Vicentino's enharmonic lute: A key to understanding his tuning system

Denzil Wraight
www.denzilwraight.com/Vicentino.pdf

Abstract

Semitone sizes Vicentino described for his *Archicembalo* (ratios 21:20 and 14:13) together exceed a major whole tone, which Maniates called a "glaring discrepancy". Vicentino gives an illustration of the frets for an enharmonic lute, previously adjudged as flawed. The fourth is not badly *divided*, but has correct major and minor tones in Roman inches to which a flawed semitone was added. The whole tone of 2½" implies that the Roman *Archicembalo's* c^1 string length is 22½" and the 21:20 ratio is an *intended* part of the scheme, expressed as 21" and 20". The enharmonic tetrachord is skillfully constructed as three overlapping series of superparticular ratios, creating arithmetical series, but not with equal diesis sizes. The *Archicembalo's* physical string lengths (in its higher-pitched version) and the interval ratios are in agreement, documenting in hidden fashion the practice and describing the theory, which is a numerological feat of great elegance. Vicentino's enharmonic is different from Zarlino's, but Vicentino's three genera do not provide mutually compatible parts of an octave division. The arithmetical division is incompatible with a practical harpsichord temperament, but the 21:20 ratio is not a "glaring discrepancy". It represents the difference between *musica speculativa* and Vicentino's compositions which were *musica prattica*.

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1. Introduction

Vicentino does not give us a detailed description in *one place* in the *L'Antica musica* of all the intervals he envisaged in his tuning system, nor is this system clearly or understandably coordinated with the tuning of his *Archicembalo*. As a result there are difficulties in resolving apparent inconsistencies in the sizes of intervals which his system reportedly used and those with which the *Archicembalo* could have been tuned. Maniates may have been the first to have focussed on these inconsistencies, calling one of them a "glaring discrepancy", where the combined semitones are too large to form even a major whole tone. This is given in reduced, tabular form below (Table 1).[1]

Table 1. The sizes of semitones used in the *Archicembalo*

Interval	Vicentino, Book V Cap. LX, fol.143v°	¼ comma meantone	Rossi 1666 31 ETS
major semitone [MS]	14:13 ratio [128.3 cents]	117.1 cents	116.1 cents
minor semitone [mS]	21:20 ratio [84.5 cents]	76.0 cents	77.5 cents
combined whole tone	[212.8 cents]	193.1 cents	193.6 cents

The third column shows the size of intervals from a practicable, ¼ comma meantone tuning (i.e. with perfect major thirds) for the *Archicembalo*. The fourth column shows Rossi's interpretation in 1666 of Vicentino's 31-note system.[2] As is well known, Rossi's 31-note equal tuning system [31 ETS] is very close to ¼ comma meantone, but both temperaments differ considerably from the ratios Vicentino gives us for the major and minor semitones (second column). Furthermore, Vicentino's combined semitones exceed the whole tone (ratio 9:8, 203.9 cents) in size. This is the problem.

It is the purpose of this article to present an explanation of the inconsistencies through the information which Vicentino's enharmonic lute division provides, shown at the end of his book as a diagram.

2. Vicentino's instructions for tuning the Archicembalo

Regarding Vicentino's description of the tuning of the *Archicembalo*, Maniates saw a "careless mistake", which we might think would question his competance or disqualify him as a reliable reporter of even his own system.[3] According to the usual translations Vicentino tells us that the lower keyboard of the *Archicembalo* is tuned "as is customary with other instruments, with the fifths and fourths somewhat *blunted* [*spontate*, i.e. both are narrower], as is done by good masters". However, when the fifths are reduced in size in a temperament, the fourths must be *increased*.

[1] Maniates, 1975, p. 344, selecting from Table 5. Rossi's cent values are those given by Maniates.
[2] Rossi, pp. 85-88. Slight differences in the diesis interval ratios can be found at the fifth decimal place, which may be due to the logarithm tables Rossi was using. Such tables became available from the 1620s.
[3] Maniates 1996, p. 332, note 26: "Vicentino's reference to blunted fourths is a careless mistake since he knew that fourths were enlarged in tempering a keyboard."

It is surprising that Maniates is the only commentator to report this discrepancy.[4] However, she also cited another instance where Vicentino described the alterations correctly.[5] Here he writes that "...*le Quarte e le Quinte di Boetio sono perfette, & quelle che noi usiamo sono un poco spontate e scarse nel accordare li strumenti*;" If we take *spontate* and *scarse* as referring respectively to the fourths and fifths, then we must allow that *spontate* should be translated as "modified" or "altered", without specifying that it be narrowed. *Scarse* then gives us the narrowed fifth. Six lines below this the matter is perfectly clear since Vicentino writes of "...*spontar una quinta e allongare una Quarta*...", so here he is obviously using *spontare* to indicate the narrowing of a fifth, and he understands that fourths have to be increased in size.

This apparent difficulty of translation and interpretation also occurs in Aaron's instructions for a tuning which has often been seen as a prescription for ¼ comma meantone with pure thirds. Here Aaron writes that "*per laquel participatione restano spuntate overo diminute le terze & seste*".[6] Again, the same "error" appears to require that both thirds and sixths be reduced.

Is it likely that both writers are in error? There is a simple way out of this difficulty if we take the texts at face value where *spontate* can mean either "increased" or "reduced", but depending on the context. This reading is not contrary to usage described in a 1612 dictionary: whereas the normal sense of *spuntare* is that something is taken away or removed (whence *blunted*), there is also the slightly poetic sense of something beginning to grow (e.g. shoots) and increase.[7] This explanation also removes some of the difficulty of interpreting Aaron's text and would explain why Vicentino could use *spontate* for the fourths and *scarse* for the fifths without any contradiction.

3. Vicentino's semitones and dieses

Of more substantial difficulty is the discrepancy between the sizes of the semitones Vicentino gives us and the *Archicembalo's* probable tuning, but it is not necessary to report his complicated theory in detail since, in describing the composition of the enharmonic genus, he leads us into the essence of the problem. The tetrachord, spanning a fourth, for the enharmonic genus is as follows:[8]

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[4] Barbieri, 2008, p. 313 in his translation gave an incorrect emphasis: "a great deal narrowed". Kaufmann, 1970, p. 85, note 9 cited Vicentino's text without seeing the difficulty, as also did Tiella 1980, p. 206. Lindley, 1974, p.150 gave us "somewhat trimmed" as a translation, and in similar vein Lindley, 1987, p. 151: "alle Quinten sollten 'etwas gestutzt [sein]' ".

[5] Maniates 1996, p. 332, note 26, reporting Vicentino in Book 1, Cap. VI [fol. 13v°, evidently the fourth line from the bottom of the page].

[6] The Italian text and an English translation is given by Lindley, 1974, pp. 142-144; this passage is on p. 144. Lindley, 1990, p.18, observed that Aaron's remark about both 3rds and 6ths being narrowed cannot be right. My discussion of this subject was hampered by the same difficulty of understanding spuntate as "blunted": Wraight 1997, Part 2, pp. 130-132.

[7] Vocabolario, p. 840: see "SPVNTARE".

[8] The form follows the first musical example of Book I, Cap. VIII, fol. 15r° and was inspired by Rasch's approach.

Table 2. Vicentino's tetrachord containing an enharmonic interval

B	Ḃ	C	E
semitone 16:15		major third 5:4	
fourth 4:3			

Since C – E is described as a major third, it is clear that the notes B - Ḃ - C must comprise the 16:15 interval of just intonation, the diatonic semitone, (111.7 cents), as Maniates inferred.[9]

Ḃ is an intermediate note, which Vicentino represents with a dot over it.[10] The size of the B - Ḃ enharmonic interval [diesis] is defined as half of a minor semitone, and Ḃ - C is equivalent in size to a minor semitone.[11] Thus, the 16:15 diatonic semitone [*semitono maggiore*] is divided into three parts.

The following size relationships obtain:

diesis [minor diesis, mD] = half minor semitone [mS]
major diesis [MD] = minor semitone [mS]
2 minor dieses [2 mD] = major diesis [MD]
minor diesis [mD] + major diesis [MD] = major semitone [*semitono maggiore*][12]

There is some potential for confusion in the terminology between the major and minor semitones comprising a whole tone, and the 16:15 diatonic semitone.

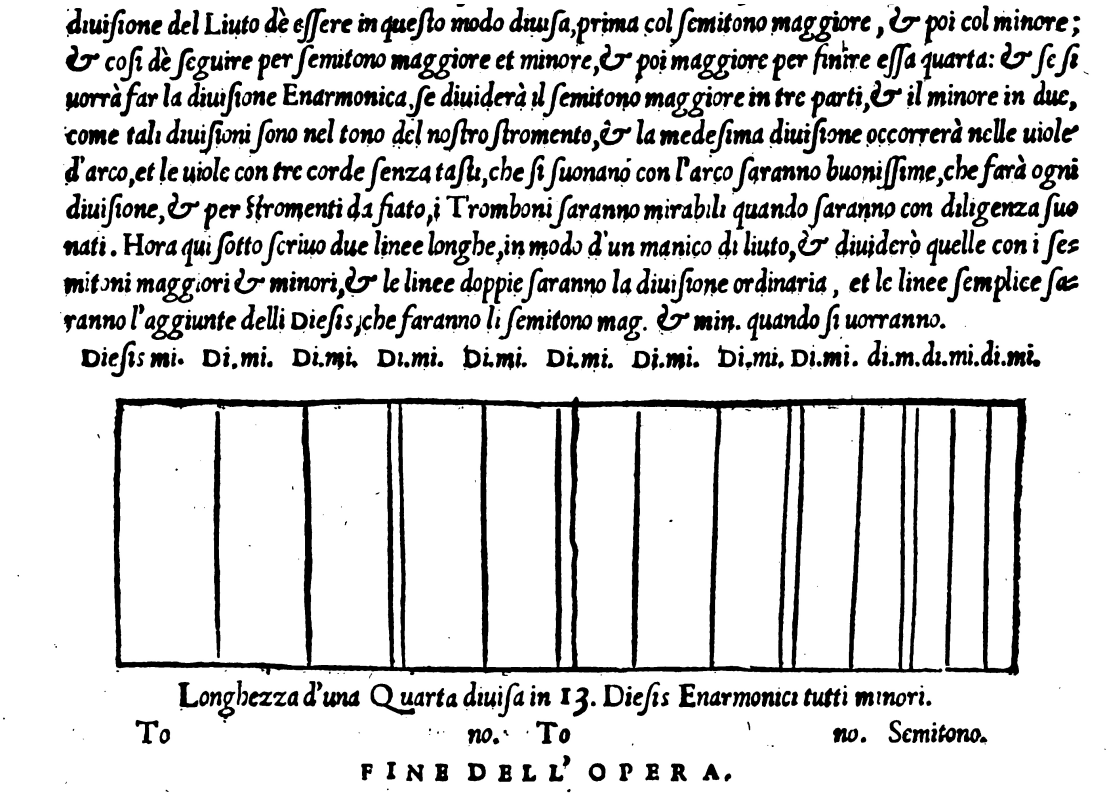
Since Vicentino tells us that his major third [*ditono*] is comprised of two tones, with the ratios 9:8 and 10:9, it appears that the sizes in the entire tetrachord could be calculated.[13]

[9] Maniates, 1993, p. 1298, found no mention of this 16:15 interval size for the semitone in Vicentino's book, and commented on Vicentino's strange omission, but she inferred the size. See also Maniates, 1996, p. xli for this same inference.
[10] Rasch's explanation, p. 41, used B+ instead of Vicentino's dot over the B.
[11] Vicentino, Book I, Cap. VIII, fol. 15r°: " Io lo do facile, accio si vegga la sua divisione, che partendosi il prattico con la voce dalla prima nota alla seconda, alzerà tanto la voce quanto sarà la metà del semitono minore: et il restante cioè dalla seconda nota alla terza, che finirà il semitono maggiore, intonerà tanta distantia di voce, quanto è un semitono minore;..." Maniates, 1996, translation p. 50: "And I present it in a simple way, to demonstrate its division. Thus, in moving from the first to the second note with the voice, the practitioner will raise his voice by the amount of one half of a minor semitone. For the remainder of the major semitone - that is from the second to the third note - he will sing the distance equivalent to a minor semitone."
[12] Vicentino, Book I, Cap. V, fol. 11r°: " Quando esso punto si ritrovirà passar per il grado del semitono minore allora sarà dimandato Diesis minore Enarmonico; che il semitono sarà di due Diesis minor Enarmonici, et quando il punto dividerà il semitono maggiore, allhora il primo Diesis Enarmonico sarà Diesis minore è il rimanente, che finirà il semitono maggiore sarà uno Diesis maggiore Enarmonico: & è di tanta longhezza, come è uno semitono minore;" Maniates, 1996, translation p. 36: "When the dot is located between the interval of a minor semitone it is called a minor enharmonic diesis, since this semitone comprises two minor enharmonic dieses. But when the dot divides the major semitone, then the first enharmonic diesis is minor and the remainder, which fills out the major semitone, is a major enharmonic diesis. The latter is the same as a minor semitone."
[13] Vicentino, ibid.: "...il nostro Dittono sarà l'interuallo di due toni, un sesquiottauo, è l'altro sessunono;..."

In his chapter on the *Archicembalo* (Table 1, column 2), Vicentino tells us that the major and minor semitones of this instrument are 14:13 and 21:20 respectively.[14] This implies that a whole tone is said to be divided into these two semitones, but, as has been observed by Maniates, this is impossible.[15] Since 14:13 = 128.3 cents and 21:20 = 84.5 cents, the sum of these intervals is 212.8 cents; the major whole tone is 203.9 cents, so something is wrong somewhere. This is what Maniates called a "glaring discrepancy".[16] Later she offered the assessment that "...Vicentino has tried to devise workable superparticular ratios from a simple concept of the natural arithmetic series."[17]

4. The enharmonic lute division

On the last page of text in his treatise, Book V, fol. 146v°, Vicentino has an illustration of frets spanning a fourth, divided into 13 enharmonic dieses. This is presented as a possible division for a lute or other stringed instrument. For many years the Bärenreiter facsimile edition of the Hannover copy was the main source for the study of Vicentino's work, but more recently digitised versions have become available from major libraries. These show that there were at least two versions of fol. 146v°. What is possibly an older version than the Hannover copy is found in a Rome copy of the 1555 printing.


diuiſione del Liuto dè eſſere in queſto modo diuiſa, prima col ſemitono maggiore, & poi col minore; & coſi dè ſeguire per ſemitono maggiore et minore, & poi maggiore per finire eſſa quarta: & ſe ſi uorrà far la diuiſione Enarmonica, ſe diuiderà il ſemitono maggiore in tre parti, & il minore in due, come tali diuiſioni ſono nel tono del noſtro ſtromento, & la medeſima diuiſione occorrerà nelle uiole d'arco, et le uiole con tre corde ſenza taſti, che ſi ſuonano con l'arco ſaranno buoniſſime, che farà ogni diuiſione, & per ſtromenti da fiato, i Tromboni ſaranno mirabili quando ſaranno con diligenza ſuonati. Hora qui ſotto ſcriuo due linee longhe, in modo d'un manico di liuto, & diuiderò quelle con i ſemitoni maggiori & minori, & le linee doppie ſaranno la diuiſione ordinaria, et le linee ſemplice ſaranno l'aggiunte delli Dieſis, che faranno li ſemitono mag. & min. quando ſi uorranno.

Dieſis mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. di.m.di.mi.di.mi.

Longhezza d'una Quarta diuiſa in 13. Dieſis Enarmonici tutti minori.

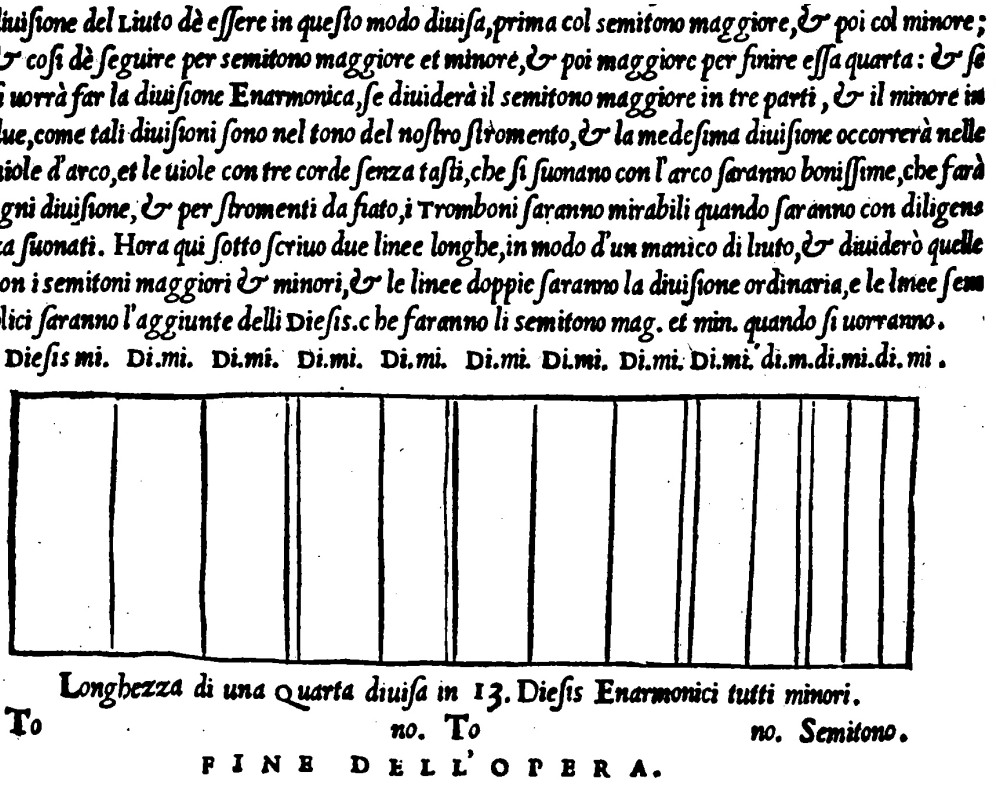
To no. To no. Semitono.

FINE DELL' OPERA.

Fig 1. Vicentino's diagram, Rome version, 1555
Source: by courtesy of Google Books

[14] Vicentino, Cap. LX, fol. 143v.
[15] Maniates, 1975, and later Rippe.
[16] Maniates, 1975, p. 344.
[17] Maniates, 1993, p.1299.

Another version (below) occurs both in copies dated 1555, and 1557.


diuiſione del Liuto dè eſſere in queſto modo diuiſa, prima col semitono maggiore, & poi col minore; & coſi dè ſeguire per semitono maggiore et minore, & poi maggiore per finire eſſa quarta: & ſe ſi uorrà far la diuiſione Enarmonica, ſe diuiderà il semitono maggiore in tre parti, & il minore in due, come tali diuiſioni ſono nel tono del noſtro ſtromento, & la medeſima diuiſione occorrerà nelle uiole d'arco, et le uiole con tre corde ſenza taſti, che ſi ſuonano con l'arco ſaranno boniſſime, che farà ogni diuiſione, & per ſtromenti da fiato, i Tromboni ſaranno mirabili quando ſaranno con diligenza ſuonati. Hora qui ſotto ſcriuo due linee longhe, in modo d'un manico di liuto, & diuiderò quelle con i semitoni maggiori & minori, & le linee doppie ſaranno la diuiſione ordinaria, e le linee ſemplici ſaranno l'aggiunte delli Dieſis. che faranno li semitono mag. et min. quando ſi uorranno.

Dieſis mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. di.m.di.mi.di.mi.

Longhezza di una Quarta diuiſa in 13. Dieſis Enarmonici tutti minori.

To no. To no. Semitono.

FINE DELL'OPERA.

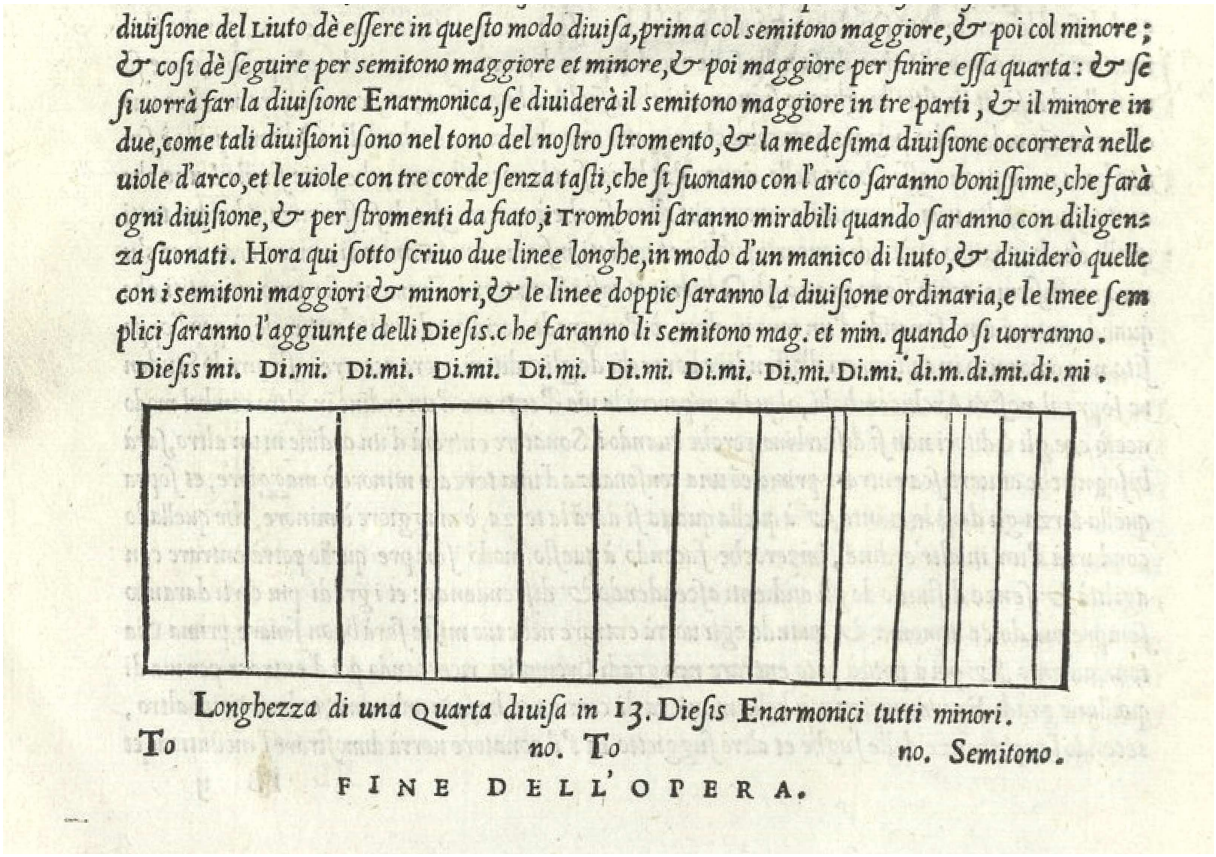
Fig. 2. Vicentino's diagram, Cremona version, 1557
Source: by courtesy of Google Books

The main difference in the Cremona version (Fig. 2) is in the re-setting of the type above the diagram (below the line "*divisione...minore;*"), which labels the diesis sizes, and below the diagram that "*Longezza d'una Quarta*..." becomes "*Longezza di una Quarta*..." [18] The words
"Tono Tono Semitono" are better aligned with the frets in the Cremona version. The printing of the lute fretting is unchanged, although variations in the amount of ink applied to the printing block, and the lack of flatness of the page during scanning, can give some versions a different appearance.

It will also be seen that either side of the second pair of double lines (counting from the left side) the spacing of the dieses is slightly different.

[18] Maniates, 1996, xxiii, compared copies of a 1555 and 1557 printing held in the Euing Library, Glasgow, and concluded incorrectly that the versions were identical.

A copy held by the Bibliothèque Nationale de France (Fig. 3, BNF first) explains this variation since this second pair of double lines is missing; there is only a single line.


diuiſione del Liuto dè eſſere in queſto modo diuiſa, prima col semitono maggiore, & poi col minore; & coſi dè ſeguire per semitono maggiore et minore, & poi maggiore per finire eſſa quarta: & ſe ſi uorrà far la diuiſione Enarmonica, ſe diuiderà il semitono maggiore in tre parti, & il minore in due, come tali diuiſioni ſono nel tono del noſtro ſtromento, & la medeſima diuiſione occorrerà nelle uiole d'arco, et le uiole con tre corde ſenza taſti, che ſi ſuonano con l'arco ſaranno boniſſime, che farà ogni diuiſione, & per ſtromenti da fiato, i Tromboni ſaranno mirabili quando ſaranno con diligenza ſuonati. Hora qui ſotto ſcriuo due linee longhe, in modo d'un manico di liuto, & diuiderò quelle con i semitoni maggiori & minori, & le linee doppie ſaranno la diuiſione ordinaria, e le linee ſemplici ſaranno l'aggiunte delli Dieſis. che faranno li semitono mag. et min. quando ſi uorranno.

Dieſis mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. Di.mi. di.m.di.mi.di.mi.

Longhezza di una Quarta diuiſa in 13. Dieſis Enarmonici tutti minori.

To no. To no. Semitono.

FINE DELL'OPERA.

Fig 3. Vicentino's diagram, BNF first version, 1555
Source: by courtesy of the Bibliothèque National de France

Examination of some other copies shows that these "errant" double lines were created by a handdrawn line in all these printings. Often the extra line is on the left (Cremona, Wolfenbüttel, Dresden, New York), sometimes it is on the right (Rome). Occasionally the line is ragged (Rome), but sometimes it is missing (BNF first, Sibley Music Library). Whether the printer added the extra line by hand, or the owners of the various copies did this has not yet been ascertained.[19]

Somewhat surprisingly, few scholars of Vicentino's works have discussed this division. Maniates published a CAD version, which is faulty, having 15 divisions instead of 13.[20] Barbieri reproduced the page in facsimile, but did not examine Vicentino's division.[21] The inaccuracy of this drawing, and lack of parallel lines, do not encourage us to believe that it could yield much useful information, and Brink, after noting that exact measurements cannot be taken, opined that it did not make any

[19] Luigi Collarile is in the process of comparing the various versions of Vicentino's book and might be able to shed light on this matter.
[20] Maniates, 1996, p. 443.
[21] Barbieri, 2002, p. 173: 'Thus all the minor semitones of Table 15 would have been bisected giving rise to octaves divided into 31 acoustically «equal fifths of a tone»'. Barbieri's asessment remains unchanged in his later book, Barbieri, 2008, pp. 41-43. Lindley, 1987, p. 150, reproduced Vicentino's monochord, but did not discuss it.

sense.[22] Like Brink, Cordes looked more deeply into the matter, testing it as a practical enharmonic lute fretting, and realised that the drawing was deficient. He decided that the exaggerated reduction at the right hand end appeared to be due to a thoughtless division of the printer.[23] This observation takes us in the correct direction.

If we take Vicentino at his word that this is the *division* of a fourth into 13 enharmonic dieses then there is indeed a problem: the whole tones are too large, at 227 and 201 cents. However, if we assume that the larger whole tone is correctly drawn for a 9:8 ratio (203.9 cents), then we find that the other tone is 185 cents, which is plausibly close to the minor whole tone (10:9, 182.4 cents), given the accuracy of the printer's work.[24]

The problem in this diagram is not with the two whole tones, but with the semitone. It is therefore is not a fourth which has been *divided*, but that two whole tones have been constructed, to which a flawed semitone has been added.

These two tones together, 9:8 + 10:9, form the major third of just intonation, so this division is not a tempered division for the keyboard, nor an equal division for fretted instruments, but a division in just intonation (whether sung or played on an instrument). As such it is incompatible with the way real lutes were tuned.[25]

[22] Brink, p. 137: "If measurements that represent the mean of the maximum and minimum lengths of the frets are taken, the ratios which result do not make any sense. These ratios and the corresponding cents values, given in Table 9 (infra p.139), do not approximate the values given by his interval ratios (Table 8, supra p. 136)." [Brink's Table 8 is the same as Table 6 in this article, but with the addition of the ratios 10:9, 9:8, 3:4 and 2:3.]. Brink went on to find a similarity to his Version 3 tuning (Table 6, p. 131), but this was based on his assumption that the entire diagram represented the interval of a fourth.
[23] Cordes, p. 67: "...die Verringerung der Abstände auf der Seite nach rechts (Richtung Corpus) ist selbstverständlich physikalisch notwendig, hier jedoch in einer Art und Weise überzeichnet, dass sie eher Folge einer unbedachten Raumeinteilung seitens des Buchdruckers zu sein scheint."
[24] Similar results are obtained if one assumes that a major third has been divided into the major and minor tones.
[25] See Cordes for a practical realisation of an equal temperament for the lute.

5. The lute division as the basis of the *Archicembalo*

Vicentino's book was printed in Rome in 1555 so the hypothesis was investigated whether the Roman foot measure might yield any clue towards understanding this division (1' = 297.9 mm; 1", i.e. 1/12 foot, = 24.825 mm). See Fig. 4, below.

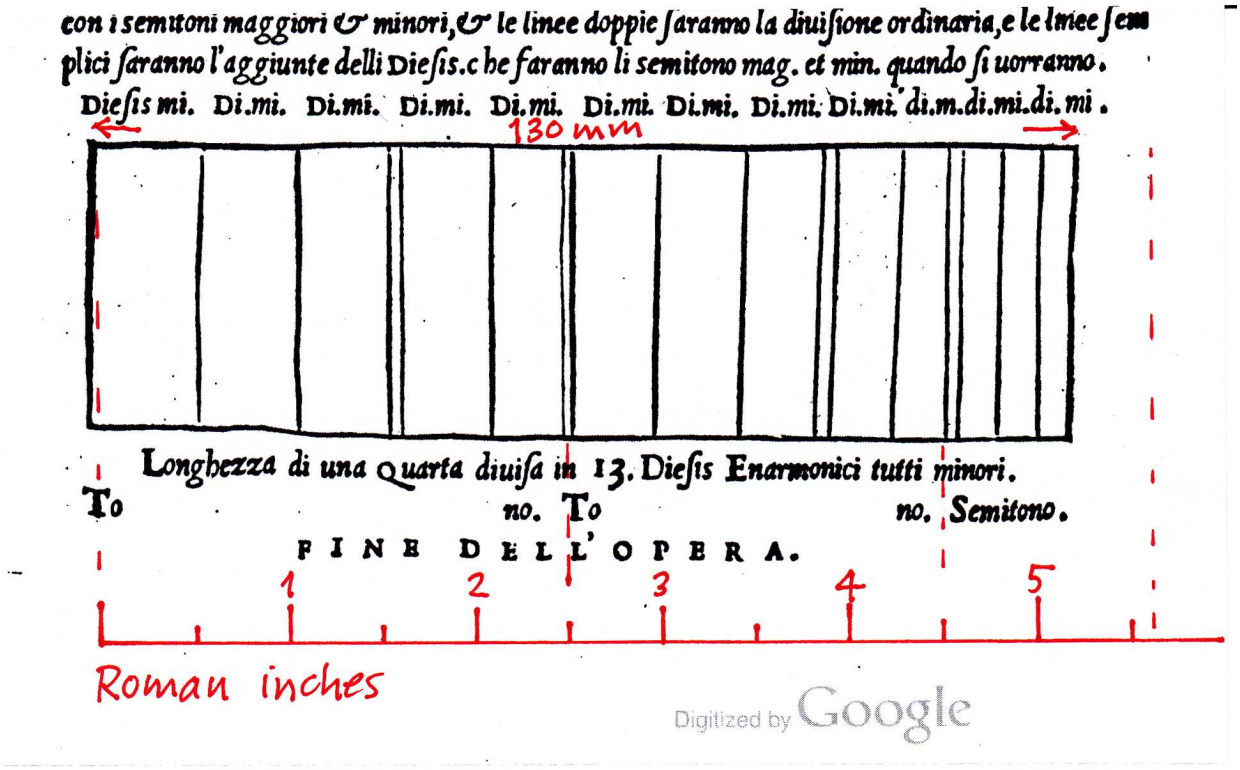


Fig. 4. The enharmonic division with Roman inches
Source: by courtesy of Google books, Cremona copy, 1557[26]

The whole tone occupies 2½" Roman inches (seen in red in Fig. 4) and the minor tone 2" (2½" to 4½" on the scale). One should understand this division as a monochord with an imaginary bridge to the right, outside of the page.[27] Since the larger whole tone (ratio 9:8) must occupy one part, with 8 parts towards the bridge, it follows that the 2½" require a total "string length" of 22½" (left end of division to the imaginary bridge). Then the line dividing the major and minor tones falls at 20", and the beginning of the minor tone (seen from the bridge side) is at 18". Of course, this ratio for the minor tone of 20:18 can be simplified to the conventional 10:9.

It is now clear that we are dealing with a distribution based on whole numbers expressed in *actual* inches rather than a mechanical division with dividers and dimensionless parts, such as was usually described in treatises on monochords.

Before examining the implications of the monochord division we can enquire whether these dimensions could be the real basis of the string lengths of the *Archicembalo*. In

[26] The lower line of the enharmonic division is 130 mm long and should print to this dimension on A4 paper with 100% set in the printer menu.
[27] Whether one imagines a second bridge at the left end of the division is immaterial to the argument at this stage since we are only dealing with string lengths from the right bridge to the left end of the *division*. This shorthand version has a long tradition: In Boethius, Book 4, 6, we have the same situation where the bridge has to be imagined by the reader. See Bower's note 51, p. 131.

Book V, Cap 2, Vicentino gives practical details for the construction of an *Archicembalo*, including a line, the length which is to be taken two times for the first string of the soprano.[28] Although it is not specified which note Vicentino intended, our knowledge of instrument making indicates that it is the c² which should be 316 mm long.[29]

If this 22½" from the monochord (= 558.6 mm) were intended for c¹, it would lead to a c² string length of 279.3 mm. This would usually imply strings of brass wire [ottone], and Vicentino even mentions such when describing Pythagoras' monochord.[30] This is plausible for a harpsichord of this period, at approximately a¹ = 415 Hz pitch. This c² length of 279.3 mm corresponds closely to a higher-pitched *Archicembalo* which Vicentino indicates would be suitable for *accompanying singing* (i.e. c² = 281 mm), since the measurements given were intended for a whole tone lower.[31] This implies that the c² = 316 mm would be 8/9 in the higher pitched instrument, or 281 mm. The c¹ = 22½" (559 mm) measurement is also found in a Roman harpsichord of the 16th century.[32] Thus, the tetrachord corresponds to the range c¹-f¹ of the *Archicembalo*, in its higher-pitched version, designed in Roman inches.[33]

Since there is a basic agreement between the instructions for the *Archicembalo* (designed for singing) and the diagram of the lute division it is probable that this division at the end of the book represented both the *theoretical* and practical basis of the *Archicembalo*, as well as the lute division.

[28] Vicentino, Book V, Cap. II, fol. 100v°. The string length is the bottom line on fol 100v°. Christopher Stembridge kindly communicated in the early 1980s that the length of the line at the bottom of 100v° in one of the British Library's copies of Vicentino's book is the same size as in the Bärenreiter facsimile edition, Vicentino R/1959. The Bayerische Staatsbibliothek later provided me with a measurement from its copy which confirmed this.

[29] See further discussion of this in the Appendix.

[30] Vicentino, Libro Della Theorica, Cap. IIII, fol. 4r°: "...una corda sonora di Nervo, ò d'ottone,..."

[31] Vicentino, Book V, Cap.II, fol. 101: "...quando [il Archicembalo] serà fatto un poco piu piccolo, acciò si possi cantare con esso, che con queste misure è un tono piu basso."

[32] A harpsichord made in 1584 by Filippo Fabri in Rome is the only usefully comparable Instrument: see Beurmann p. 32-33. Beurmann gives c¹ = 559 mm, c² = 286 mm, although he considered the soundboard to be a later replacement. The activity of a Francesco Fabri in Rome is documented by Barbieri, 1989, pp. 150-151. Four instruments made by Cristofori in Florence for the same pitch show a range of string lengths from 280 mm to 288 mm and thereby confirm that 281 mm can be seen to be within the manufacturing tolerance when c¹ = 559 mm and c² = nominally 279.5 mm. See Wraight 1997, Part II, pp. 111-114, W85, W89, W679, W90.

[33] There are other measurements in the *Achicembalo* drawings which can be expressed in round numbers of Roman inches: see the Appendix.

6. Analysis of the enharmonic lute division

Although it was the Roman inch which led to this line of analysis, no foot measure is necessary in order to obtain these intervals since they are defined by numerical relationships. We should recall the significance that the practical monochord had for the musician in thinking about intervals or giving instruction in the fundamentals of music.[34] In 1529, when Vicentino was 11, Fogliano published a perspective illustration of a "researcher" at his monochord.[35]

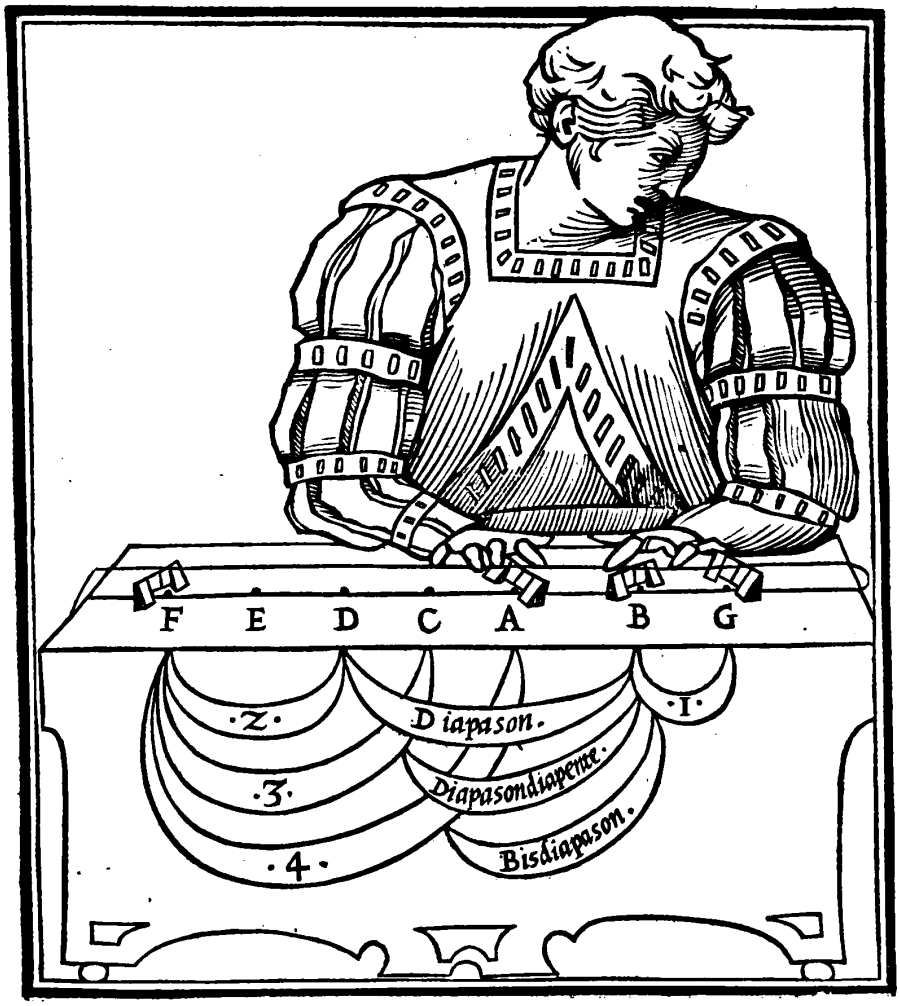


Fig. 5. Fogliano monochord, fol. XIIv°
Source: by courtesy of Google Books

[34] Buehler-McWilliams and Murray.
[35] Fogliano fols. XIIv°, XIII and XIIIv°.

In 1558 Zarlino reminded us of this way of thinking when he presented a monochord on which a string length with two bridges is clearly shown.[36]

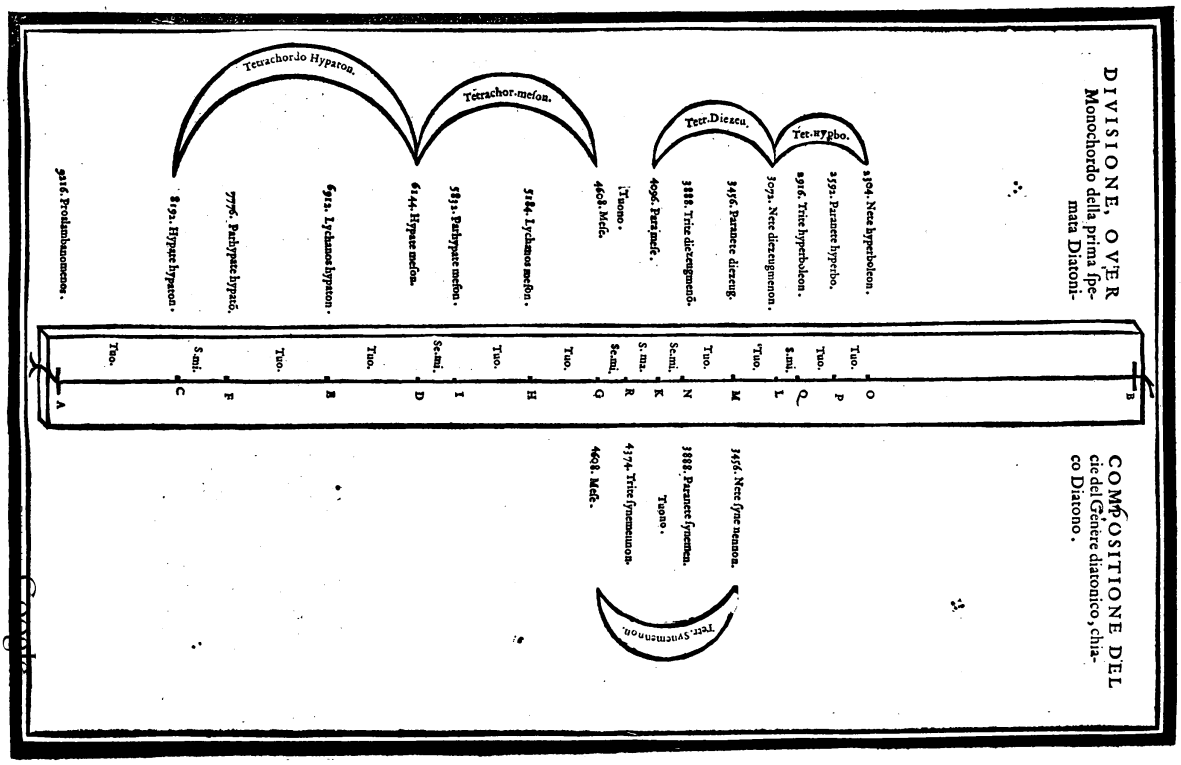


Fig. 6. Zarlino, 1558, p. 100 (aligned horizontally with Fogliano's monochord)
Source: by courtesy of Google Books

[36] Zarlino, 1558, Cap. 28, p. 100 "DIVISIONE, OVER COMPOSITIONE DEL Monochordo".

Fogliano also published an octave division (Fig. 7) containing 4-figure numbers, because he showed chromatic degrees of each note.

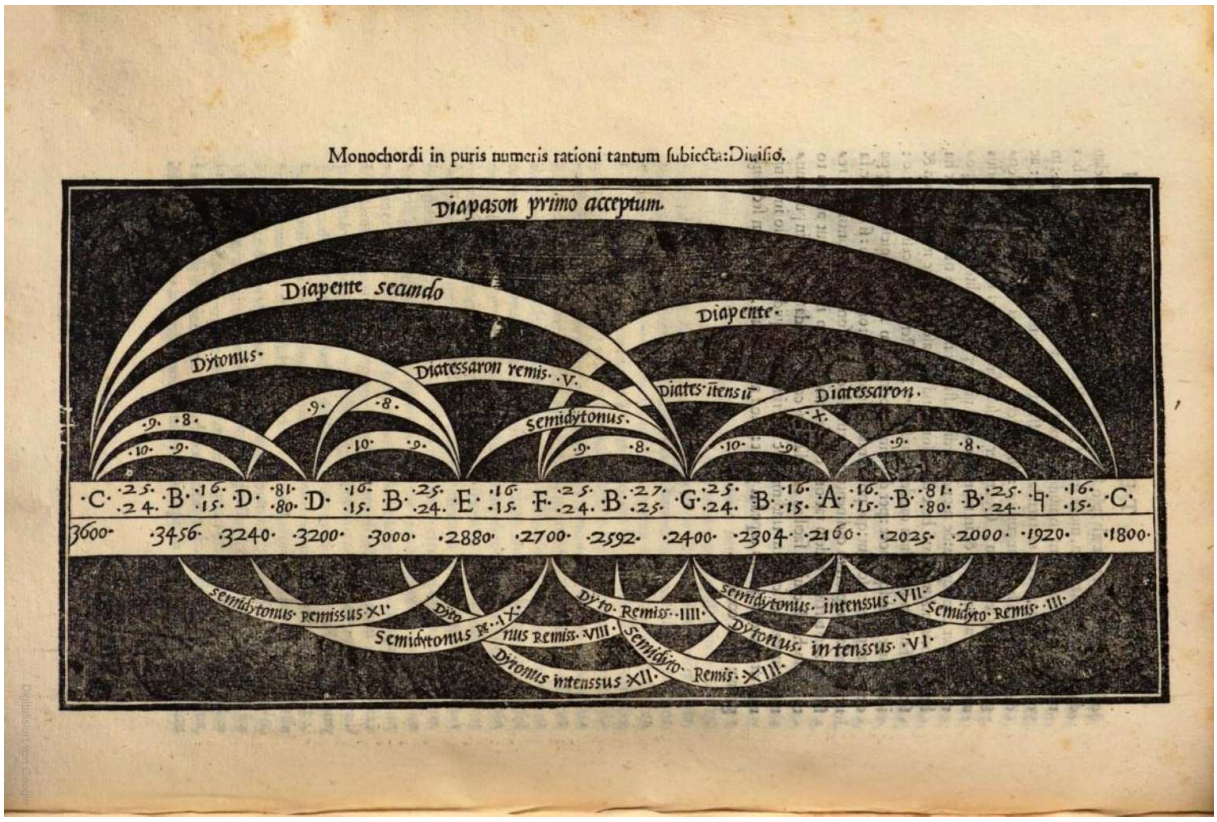


Fig. 7. Fogliano's octave division, fol. XXXIIIIv°
Source: by courtesy of Google books

Barbieri published a monochord drawn by Vincenzo Colombi, the Venetian organ builder who made an *Arciorgano* for Vicentino, on which the diatonic notes are described by the same values (but as 3-figures), suggesting the acquaintance that practitioners probably had with such theoretical works.[37]

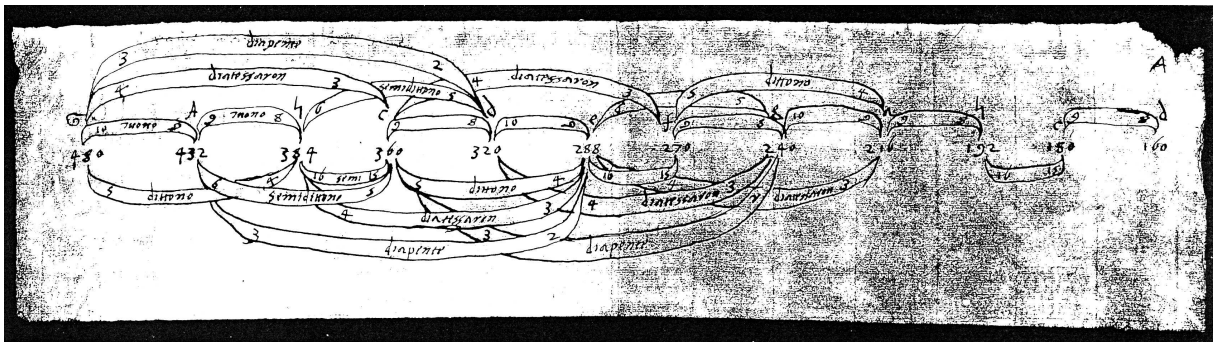


Fig. 8. Colombi's octave division
Source: by courtesy of Patrizio Barbieri (2002)

Colombi's sizes correspond to Fogliano's, albeit at 1/10 size, but several of the sizes would be found as whole numbers on the measuring stick of any artisan, virtually

[37] Barbieri, 2002, p. 160, but kindly made available to me by Barbieri before its publication in 2002.

regardless of which foot measure he used, e.g. C = 36", D = 32", F = 27". Thus, much of what we might now think involves calculation could simply be taken from any foot measure by an organ builder, at least for simpler ratios such as the fourth and fifth, but before we try and find a foot measure behind every monochord we should be aware of other traditions.[38]

The large, and for the modern reader probably strange, number of 2304 for the starting note in Boethius's three tetrachords (the G"B" = G♯ of Fogliano's monochord) is, as Barbera has explained, the least common multiple of 9 and 256, stemming from the ratios 9:8 and 256:243 found in the diatonic tetrachord.[39] A major tone lower than 2304 yields the figure 2592, also found in Fogliano's monochord (F "B" = F♯), but it is also the *starting* point (x16 = 41472) on A of Rossi's 31-note equal temperament interpretation (published in 1666) for Vicentino's *Archicembalo*.[40] Such is the span of tradition among theoreticians from the sixth century (AD) to the 17th century.

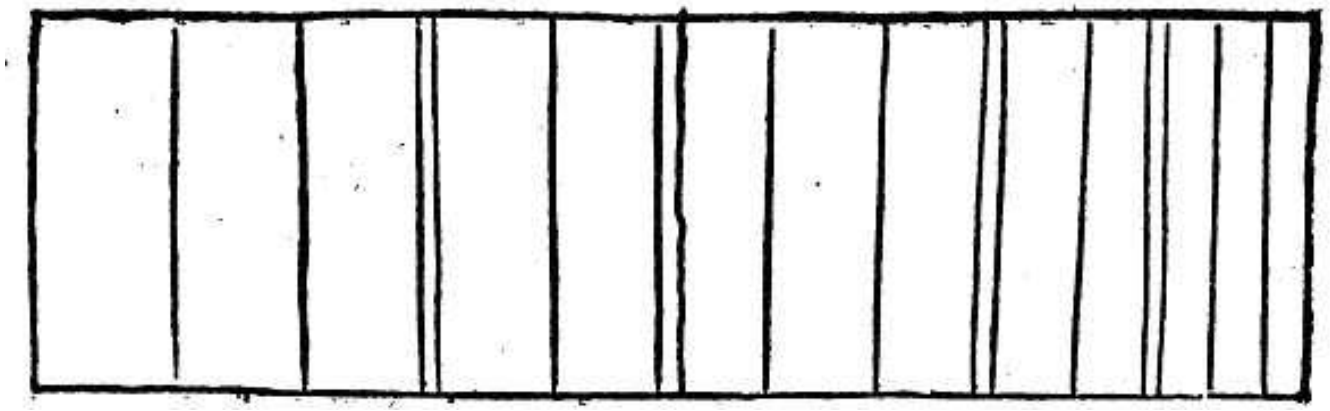
When we consider Vicentino's enharmonic lute diagram as the representation of a monochord we should think of the originating point of the "string" as being at the right hand bridge, which is not shown on Vicentino's division.

In her translation of Vicentino's book Maniates gives the ratios *numerically* in the following form, 8:7 or 21:20.[41] However, in Vicentino's text we find " 7. à 8." and "20. à 21." The significant difference is not that we now use a colon to indicate a ratio, but in the *order* of the numbers and the ".".

Thus "7. à 8." means "7 steps from the bridge, then 8 steps from the bridge". The "." may seem insignificant, but it reminds us of the numerous monochord instructions found in manuscripts, made with dividers, where 7 steps would be shown as "7.", the "." representing the point mark made by the divider.[42]

We have established that in Vicentino's enhamonic division the longest string (left end of division to the "invisible" bridge) is 22½". The line dividing the major and minor tones falls at 20", and the beginning of the minor tone (seen from the bridge side) is at 18" (Table 3 below).[43]

[38] Fogliano, fols. XXXIIIIv° and XXXVIIIr°.
[39] Boethius, Book IV, 318-324 (Bower, pp. 130-135). This useful insight is buried in Barbera, p. 322, note 33. Readers acquainted with Boethius will be less surprised by the number 2304.
[40] Fogliano, f. XXXIIIv° and Rossi, p. 86.
[41] Maniates, 1996, pp. 436 and 433 respectively. I use either form in the text, according as is appropriate to the argument.
[42] A well-known example is found in Arnaut de Zwolle's manuscript, fol. 129r°, describing the construction of a clavichord.
[43] Note that the inch scale is not linear in Table 3 because equal spaces have been given between the numbers below the inch scale.



Vicentino's diagram, Rome version, 1555
Source: by courtesy of Google Books

Table 3. Vicentino's enharmonic tetrachord derived from the enharmonic lute fretting

[MS = Major Semitone, mS = minor Semitone, MD = Major Diesis, mD = minor Diesis]

fourth 4:3 498.04 cents [c]												
Major third 5:4 [20:16] 386.31 cents										Semitone 16:15 111.73 cents		
Major Tone 9:8 203.91 cents					minor Tone 10:9 182.45 cents							
MS 15:14 119.44 c			mS 21:20 84.47 c		MS 50:47 107.12 c			mS 47:45 75.28 c		MD 24:23 73.68 c		mD 46:45 38.05c
					25:24 70.67 c							
38.91c	39.80c	40.74c	41.72c	42.75c	34.98c	35.70c	36.45c	37.23c	38.05c	36.45c	37.23c	38.05c

22 1/2" **21"** **20"** **18"** **17 5/8"** **16 7/8"**

45 **44** **43** **42** **41** **40**

50 **49** **48** **47** **46** **45**

48 **47** **46** **45**

In order to examine how the lute division is constructed we double the inch measurements in order to remove the half inch at 22½" = 45. Then we have the following sequence for the major whole tone: 45 44 43 42 41 40 (Table 3 above, under the inch measurements):

Vicentino has divided his major whole tone into five parts and taken two for the minor semitone (mS, 42:40, simplified to 21:20) and leaving three for the major semitone (MS, 45:42). Thus, Vicentino's system of a whole tone divided into 5 parts is nothing more than an arithmetic division of dimensionless numbers, but which also correspond to *string lengths*. According to this *method*, each of the dieses will have a slightly different size. This variation in size of the dieses can be clearly seen in the bottom row of Table 3, the size becoming larger as one reads to the right, from 38.91 to 42.75 cents.

In fact we could have derived the 21:20 ratio for the minor semitone simply from knowing that the major tone (9:8, or 22½" : 20") is to be divided into 5 parts, with an arithmetical division, but it is only the lute division which reveals that an *arithmetical* division is involved.

Zarlino took Vincenzo Galilei to task for describing the 2/7 comma tuning system using an arithmetical division of the comma, even though the error would have been miniscule (maximally ±0.0164 cents).[44] Rossi argued nearly a century later that such errors can be neglected.[45] As seen above, in Vicentino's lute division the arithmetical method leads to interval size variations which are anything but trivial: the size of the diesis ranges from 34.98 to 42.75 cents, a difference of 7.77 cents, or approximately ⅓ syntonic comma (ratio 81:80, or 21.5 cents). The tempering of the fifth in ¼ comma meantone is 5.4 cents. What we should think of this will be discussed later.

It is clear from Vicentino's theory, and his division procedure shown in Table 3, that the major semitone (MS in the major tone) should have the ratio 45:42, which can be simplified to 15:14. Can it be that Vicentino slipped down the series of superparticular ratios (15:14, 14:13, 13:12 etc.) and gave out 14:13 in error as his major semitone (see Table 1) whereas his division method clearly leads us to 15:14? It seems that some sort of error is involved because 15:14 yields the correct ratio of 1½ mS = 1 MS which fits the entire scheme.[46]

What happens to the semitone sizes in the *minor* tone? If Vicentino were to divide the minor tone in a similar fashion into 5 parts, as his diagram indicates, then the sizes 20" and 18" need to be multiplied by 2½ so that there are six whole numbers

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[44] Zarlino, Sopplimenti, 1589, pp. 194-197; see especially lines 21-23, p.195: "le due parti dq. & qp. scemate dalla Quinta, non esse di quella quantità, che sono le aggiunte cl. & lm. d'alla Quarta".

[45] Rossi, pp. 58-59. For Zarlino it was apparently the principle which was involved, since exact (geometrical) methods were known and described by him in Le Istitutioni Harmoniche, 1558, Part 1, Cap. 37, pp. 47-49. The graphical method with a mesolabium, was described in Cap. 25, pp. 94-96. It was possibly also to punish "il caro Discepolo" (the dear disciple, i.e. Galilei), who had published a description of Zarlino's 2/7 tuning, as if it were his own. Lindley referred briefly to this dispute between Zarlino and Galilei, but Celhoffer has analysed the issue clearly and established that Zarlino was not himself advocating arithmetical division.

[46] An alternative explanation is that the difference between the major tone and the interval 14:13 is 75.6 cents, which is very close to the size of a chromatic semitone in ¼ comma meantone, which is a practicable tuning of the period. This explanation is attractive, but incompatible with Vicentino's designation of 20:21 (84.5 cents) for the minor semitone in his *Archicembalo*.

spanning the interval, i.e. 50 to 45 (in Table 3, 2nd row below the inch measurements).

50 49 48 47 46 45

Then 3 parts form the major semitone, ratio 50:47 (107.1 cents), and 2 parts will form the minor semitone: 47:45 (75.3 cents). Of course, in this fashion the major and minor semitones in the whole tones cannot have the same size. Furthermore, their size *relationship* (MS:mS) is not even the same, their being formed with an arithmetical division.[47]

The diatonic semitone (16:15) in Vicentino's diagram is defective and calculation shows that he would have needed a string length of 16⅞" for the semitone to bridge, or 1⅛" beyond the minor tone. The division should have been 5½" wide (139 mm), as shown by the vertical, red hatched line (Fig. 4), and there would even have been space for it on the page. We cannot tell what went wrong between Vicentino's copy and the printer's page, but this sort of problem was not limited to the Renaissance.[48]

Given Vicentino's statement that 1x major diesis = 2x minor dieses it is not difficult to uncover the intention in this part of the monochord. If working from the Roman inches 18" to 16⅞" we multiply by 8/3 to remove the fractions. Alternatively, knowing that the interval represented here is the *semitono maggiore* with ratio 16:15, we multiply by 3. By either route we have the number series:

48 47 46 45

which encompasses 3 parts. The minor diesis 46:45 is then 38.05 cents, the major diesis 24:23 [48:46] or 73.68 cents.

The *multiplication* of the ratio 16:15 by 3 effects a *division* into three parts. In similar fashion a division of the whole tone into 5 parts is accomplished thus:
9:8 x 5 = 45:40 with the intervening parts being 45 44 43 42 41 40
and
10:9 x 5 = 50:45 with the intervening parts being 50 49 48 47 46 45

These arithmetical manipulations were known in Antiquity and Ptolemy used such a procedure to construct his even diatonic tetrachord 9:10 10:11 11:12 (Table 4).

Thus, we have re-constructed the diatonic and enharmonic tetrachords Vicentino had conceived from the inch measurements, but also shown the simple numerical procedures involved. Moreover we have seen that the allegedly discrepant 21:20 ratio actually has a calculated and ordered place in Vicentino's tuning system.

[47] 128.3 cents : 84.5 cents = 1.52; 107.1 cents : 75.3 cents = 1.42.
[48] I can confirm this from two of my own publications: despite clear instructions, editors and printers changed the sizes without realising what errors they were introducing.

7. Greek sources of Vicentino's tetrachords

Maniates suggested that in order to create his own tuning system, Vicentino had probably drawn on Fogliano's work.[49] In an earlier publication she referred to, and listed, the various intervals Vicentino names from his system, which, according to her, are to be found in Ptolemy's diatonic syntonon, including the just fifth, minor sixth, and major sixth.[50]

Ptolemy's diatonic syntonon is introduced as a *tetrachord* in Harmonics, Book I and therefore does not contain any intervals larger than the fourth.[51] In naming the intervals larger than a fourth, Maniates has referred to Ptolemy's diatonic syntonon *monochord* with the range of an octave.[52] This was an *octave* division, without specification of note names, not a tetrachord.[53] Gaffurius (writing in Latin) re-introduced Ptolemy's diatonic syntonon tetrachord in 1518 to Renaissance scholars who did not read Greek, but it was not until 1558 that it was incorporated into an octave division by Zarlino.[54]

Since Vicentino has presented us with a *tetrachord* for the lute division, the approach will be pursued here to examine what Greek tetrachords might tell us about Vicentino's tuning system, rather than investigate the scanty material of *octave* divisions for a keyboard instrument. Crocker commented on the preëminence of the tetrachord since, in his analysis, the Pythagoreans projected the 8:9 tone "inside the fourth", rather than dividing the octave by a cycle of fifths.[55]

The source Vicentino cites most often for Greek musical history is Boethius, a Roman scholar, 480-524 AD, who could read Greek and had access to Greek manuscripts, some of which no longer survive. When we consider the possible sources for these tetrachords we find that one of them could stem from Archytas and was described by Boethius, but another can only have been taken from Ptolemy. Boethius described the headings of chapters 20-23 in Book V on Ptolemy's tetrachords, but the content was missing in the Boethian manuscripts which survived in Vicentino's time, and we have not yet re-discovered them.

Only when a translation was made from Ptolemy's Greek in 1499 by Nicolò Leoniceno (1428-1524) did Ptolemy's information become available to those unable to read much Greek, which (according to Maniates) included Vicentino, Gaffurio, and

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[49] Maniates, 1996, xxxii:

[50] Maniates, 1993, p. 1298 and Table 5. Here I follow Maniates' own name for Ptolemy's division. The diatonic syntonon is called the "intense diatonic" by Solomon, and appears on p. 103 in his complete English translation of Ptolemy's Harmonics.

[51] Solomon, p. 52, [36.35]. Gaffurius published this diatonic genus, Liber Secundus, Cap. 19, fol. XLIII, albeit with different numbering: 96 90 80 72.

[52] Maniates, 1993, p. 1298.

[53] Ptolemy, Book II, [72.6]-[74.4]. This "intense diatonic" is given by Solomon, p.103, as a vertical column of figures from 120 to 60, including six other versions of diatonic octave divisions. Barbour gives Ptolemy's diatonic syntonon as Table 23, p. 20, with the order of intervals reversed (left to right), as if Ptolemy's larger numbers were higher frequencies. We might think that the numbers are probably equivalent to string lengths, but Barbera, p.320, note 2, records (citing van der Waerden) that the Pythagoreans occasionally used higher numbers for the higher pitch.

[54] Gaffurius, Book Two, Cap. 19, fol. XLIII, Miller, p. 110. Zarlino, 1558, p. 122. Gaffurius, Book Two, Cap. 32, fols. LVIIr°-LVIIIv°, published seven octave species, but the intense diatonic was not among them. Miller's translation conveniently adds the ratios formed by the numbers in the tetrachord.

[55] Crocker, p. 197.

Zarlino.[56] Gaffurio (1451-1522) published his *De harmonia musicorum* as a book in 1518 and thereby made public Ptolemy's versions of the three genera, although Palisca has shown that this work was already completed as a manuscript in 1500.[57] Maniates considered it unlikely that Vicentino had access through Gian Giorgio Trissino (1478-1550) to Leoniceno's translation.[58] In Maniates' view Vicentino relied instead on Fogliano's "attempt" (her expression) to turn Ptolemy's diatonic syntonon into a just intonation keyboard division.[59] However, according to Palisca there is no clear indication that Fogliano used Greek sources in order to construct his just intonation monochord.[60] What can the Greek tetrachords teach us about Vicentino's sources?

[56] Maniates, 1996, xxxii, citing Palisca, p. 111.
[57] Palisca, p. 201.
[58] Maniates, 1996, xxx-xxxii.
[59] Maniates 1993, p. 1300. The expression "attempt" casts at least doubt on its success, but Fogliano's monochord is in no way defective. Here she appears to have relied on an assessment of Vicentino's knowledge of Ptolemy; Palisca, p. 253, expresses himself in almost identical vein. Fogliano's divisions are at XXXIIII and XXXVIII.
[60] Accordinging to Palisca, p. 241, Fogliano was probably aware of the similarities and differences regarding Ptolemy's and Ramos' tetrachords, but he did not name them.

Table 4. Tetrachords from Archytas to Zarlino

GENUS[61]	ratio cents	ratio cents	ratio cents	Archytas[62] 428-347 BC	Didymus[63] 63 BC- 10 AD	Ptolemy fl. 141 AD	Boethius 480-524 AD	Gaffurio 1518 Book II[64]	Zarlino 1558
soft diatonic	21:20 84.5	10:9 182.4	8:7 231.2			[36.35]		XLIIv° (97)	diatonico molle p.106[65]
tonic diatonic	28:27 63	8:7 231.2	9:8 203.9	[31.15]			Book V,17	XLIIIv° (99)	diatonico toniaco p.107
intense diatonic	16:15 111.7	10:9 182.4	9:8 203.9		[72.11][66]			XXXVIIIv° -XXXIX (89-90)	
intense diatonic	16:15 111.7	9:8 203.9	10:9 182.4			[36.35]		XLIII (98)	diatonico sintono p.83 [67]
even diatonic	12:11 150.6	11:10 165	10:9 182.4			[72.11]		XLIIII (100)	
intense chromatic	22:21 80.5	12:11 150.6	7:6 266.9			[35.7]		XLII (96)	chromatico incitato p.114
chromatic	28:27 63	243:224 140.9	32:27 294.1	[31.15]			Book V,17		
chromatic	16:15 111.7	25:24 70.67	6:5 315.6		[71.7]			XXXVIIIv° -XXXIX (89-90)	p.138 (not named)
soft chromatic	28:27 63	15:14 119.5	6:5 315.6			[35.7]		XLI (94)	chromatico molle p.113
enharmonic	32:31 55.0	31:30 56.8	5:4 386.3		[70.10]			XXXVIIIv° -XXXIX (89-90)	
enharmonic	28:27 63	36:35 48.8	5:4 386.3	[31.15]			Book V,17		
enharmonic	46:45 38.1	24:23 73.7	5:4 386.3			[35.7]		XLv° (93)	enharmonico Tolomeo p. 85 [68]
enharmonic	25:24 70.7	128:125 41.1	5:4 386.3						p.140 [69]

This presentation of tetrachords in Table 4, which could have supplied elements of the enharmonic lute division, also shows their origin in chronological order from left to right, and indicates that tetrachords from Didymus and Ptolemy could not have been known to Renaissance scholars through Boetius. Vicentino states that none of the genera he uses are similar to those *written* by Boethius, and thereby allows the

[61] The translations of the genera in to English are those given by Solomon.
[62] Archytas is only known through Ptolemy, hence the [...] which refer to the numbering in Ptolemy's Harmonics. Archytas is further relayed by Boethius' paraphrasing of Ptolemy. This information also appears in Gaffurius Liber secundus fol. XXXIXv°, in a clear tabular form. Miller p. 103.
[63] Didymus' tetrachords were also transmitted through Ptolemy, hence the numbering in [...] brackets.
[64] The page numbers in (...) brackets are those of the cited PDF file from the copy held in the Bibliotheca Casanatense, Rome.
[65] Zarlino shows this in an octave monochord
[66] Ptolemy, Harmonics, Book 2, [72.11]. Solomon, p. 101. The intervals are identical with Ptolemy's intense diatonic, but the positions of the ratios 9:8 and 10:9 are exchanged. Zarlino does not show this version.
[67] Zarlino, 1558, p. 83 for the tetrachord, which is called "Il Sintono, overo Incitato". On p. 122 Zarlino gives the diatonico sintono in an octave monochord.
[68] Zarlino, 1558, develops the enharmonic tetrachord to an octave monochord on p. 117.
[69] Chalmers, p. 171, tetrachord no. 109 with these intervals is attributed to Salinas, 1577. Zarlino would appear to have established a valid, prior claim to this tetrachord.

possibility that they were not found there.[70] In Boethius' collection were six by Aristoxenus and three by Archytas, but only one of Archytas' tetrachords could have been of some use to Vicentino. This is the tonic diatonic tetrachord, but it occupies only a minor place in the explanation.

It is Didymus' intense diatonic tetrachord which is identical with the diatonic components in Vicentino's lute division. Although Ptolemy's intense diatonic (diatonic syntonon) has the same ratios, Ptolemy places the 9:8 major tone adjacent to the 16:15 semitone, which has a quite different effect in Vicentino's monochord, as will be explained later.[71] Didymus' tetrachords were described and discussed by Gaffurius.[72]

Ptolemy's enharmonic tetrachord is identical with Vicentino's enharmonic. This was published by Gaffurius in 1518, together with the calculations (also shown p.17 above) for dividing the 16:15 semitone into three parts, so no *invention* of this was required on Vicentino's part.[73]

Thus, if knowledge of Greek tetrachords was required for Vicentino in order to derive his system, he need not have obtained Leoniceno's translation; reading Gaffurius' 1518 publication would have sufficed.

8. The enharmonic tetrachord is the key to Vicentino's system

What is striking in Table 4 is that one of Ptolemy's enharmonic tetrachords, first re-published by Gaffurius in 1518, and thereby made readily accessible to Renaissance scholars, is contained in Vicentino's lute division. Moreover, no other ancient source reported by Boethius used these ratios for constructing the enharmonic genus.

In his exposition of the enharmonic genus, Vicentino writes that "...in moving from the first to the second note with the voice, the practitioner will raise his voice by one half of a minor semitone..."[74] The minor semitone in this example (i.e. of the minor tone in Table 3) is the interval 47:45. One "half" of this by Vicentino's arithmetical method is 46:45. This 46:45 ratio also defines the smallest, *designated* interval in the enharmonic tetrachord, Vicentino's *diesis*. Thus, two *dieses* form a minor semitone, just as Vicentino describes.[75]

In this way we have the 2:1 equivalence of interval sizes; of course, it can only be approximately correct since it results from Vicentino's use of the arithmetical mean. Vicentino described the result, but he did not explain the method by which it was reached, which his lute division now reveals.

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[70] Vicentino, Book I, Cap. VII, fol. 14r° "...che scrive Boetio". It is unclear whether he means those which Boethius described, or whether those of Aristoxenus and Archytas (reported by Boethius) are also included.

[71] Barbour, p. 21, observes that "Didymus' arrangement is the more logical for constructing a monochord; Ptolemy's in terms of the harmonic series."

[72] Gaffurius, Liber Secundus, XXXVIIIv°-XXXIX (PDF 89-90), Miller p. 102, who incorrectly cites Ptolemy II:32 for the comments on Didymus; the correct citation is II.13.

[73] Gaffurius, Liber Secundus, XLv° [PDF 93], Miller p. 105.

[74] Vicentino, Cap 8, fol. 15, Maniates, 1996, translation p. 50.

[75] Ibid. It is not the smallest interval *per se*, since this is the 50:49 ratio (34.98 cents) in the Major Semitone of the minor Tone: see Table 3.

The structure of the enharmonic tetrachord also explains a curiosity of Vicentino's lute division. Vicentino has placed *both* minor semitones in the *higher* position within the tone (mS: 84.47 cents or 21:20 in the major tone; 75.28 cents or 47:45 in the minor tone, Table 3). The mS is usually found in the *lower* position in the major tone (e.g. c♯, f♯, and g♯). This is also the case in Fogliano's division (Fig. 7 above) where the minor semitone between C and "B" [C♯] has the ratio 25:24 (i.e. 50:48 simplified, = 70.67 cents). Since an octave is comprised of two tetrachords connected by a whole tone, this tetrachord represents the range C-F or G-C, but with d♭ and a♭ instead of c♯ and g♯. Of course, the mS in the minor tone (10:9) represents the usual e♭ or b♭.

This placement of the mS in the minor tone is required in the higher position if the 46:45 diesis is to be "half" of the 47:46:45 *arithmetical* series, which forms the minor semitone (see Table 3). The 16:15 semitone [48-47-46-45] and the preceeding three sections (i.e. toward the left) are identical. Had Vicentino placed the minor semitone in the lower position in the minor tone, i.e. with the ratio 50:48 (= ratio 25:24), then this cunning numerological construction with two identical sequences of epimores, where 2x mD = mS, would not work.[76] It is an elegant piece of numerical alignment, which probably had considerable attraction for the inventor, whether Vicentino or the unknown *Archicembalo* builder. Of course, it is only numerology: the mD cannot be exactly half of the mS with an arithmetical division.

This explains why Vicentino's minor tone should be placed adjacent to the 16:15 semitone: in order that the repeated numerical series is possible. As Table 4 shows us, Ptolemy's version of the intense diatonic tetrachord has the 9:8 major tone adjacent to the 16:15 semitone, which would not serve Vicentino's purpose.

Seen also from the general structure of Vicentino's system, if the 16:15 semitone (111.7 cents) is to be divided into 3 parts, then for a whole tone comprised of 5 *equal* parts the 10:9 minor tone is the more "natural" companion, even when an exact match is not possible: 5/3 x 111.7 cents = 186.2 cents, and the minor tone is 182.5 cents.

We cannot discern Vicentino's exact starting point, but if he "copied" any Greek diatonic tetrachord then Table 4 would indicate it was Didymus' intense diatonic, unknown to Renaissance scholars unless they could read the Greek of Ptolemy's Harmonics, but which became accessible through Gaffurius.

On Vicentino's own account he took two sizes of the whole tone (i.e. 9:8 + 10:9 = 5:4) and not two 9:8 tones (= 81:64, as in Pythagorean tuning) in order to create a "greater abundance of steps, consonances and harmony than did the ancients".[77] Vicentino requires the 5:4 major third as part of his (and Ptolemy's) enharmonic tetrachord, which explains the deeper meaning behind his "greater abundance of steps". In this respect he was also following the *moderna prattica* in rejecting the older Pythagorean tuning with its 81:64 major third and favouring the 5:4 major third which had by his time acquired a consonant status, at least in performed music.

[76] The term "epimore", signifying a superparticular ratio, has apparently now gone out of fashion, but was used by Crocker and Barbera without explanation of its meaning.
[77] Vicentino, fol. 15r°, Maniates, 1996, p. 49.

Fogliano's division is one of the early recorded studies which formulated this position through just intonation, and may have influenced Vicentino, as Maniates has suggested.[78] However, Fogliano's division does not supply an explanation of Vicentino's enharmonic, so this could have come from Ptolemy, most likely through Gaffurius who delivered a blueprint for the division of the *semitono maggiore*. Maniates detected 33 "allusions" to Gaffurius work, but found only one acknowledgment by Vicentino, thereby indicating that Vicentino mostly did not reveal his sources.[79]

This arithmetic division for the lute cannot provide *equally-sized* dieses, as required in a temperament, but there is one fortuitous coincidence: we can note that the 45:44 interval size (22½":22") at the lowest end of the lute division with its 38.91 cent size is a remarkably good approximation to 31-note equal temperament with its 38.71 cents, the difference being only 0.2 cents. (in ¼ comma meantone this interval is 41.06 cents).

One might wonder that Vicentino did not declare the 47:45 ratio of his minor semitone (75.28 cents) as the semitone size of his *Archicembalo*, but instead the 21:20 ratio (84.47 cents). Instead of a "glaring discrepancy" he would have achieved a good approximation to 77.4 cents, i.e. 2 parts of 31-note equal temperament, on which commentators might have congratulated him. However, the 21:20 ratio is, as noted above, two parts of a division of the whole tone into five parts by the arithmetical method. Thus, Vicentino's priority was to construct a theoretical system based on whole number ratios (mostly superparticular), not to devise a tuning scheme for the *Archicembalo*. That these superparticular ratios also coincide with the string lengths in Roman inches of the *Archicembalo* scaling is the crowning glory of his hidden scheme.

9. Vicentino's chromatic tetrachord

This examination of the enharmonic tetrachord shows how skillfully Vicentino's tuning system was constructed, and now enables us to examine the chromatic tetrachord. At first glance it appears as if Vicentino's chromatic tetrachord is based on Didymus' chromatic since Vicentino's lute division also placed the 16:15 semitone at one end of the tetrachord, but Vicentino's should be read like a monochord, with the shorter "strings" at the right.[80]

Table 5.

trihemitone	semitone	semitone
6:5 315.6 cents	25:24 70.67 cents	16:15 111.7 cents

Vicentino does not give us much information about his chromatic tetrachord, but it is enough: he states that the third step (right hand end) is the same semitone

[78] Maniates, 1996, xxvi.
[79] Maniates, 1996, xxvi.
[80] Vicentino, Libro Della Theorica, Cap. 6, fol. 4v°. Didymus' chromatic tetrachord is presented in Ptolemy [71.7], Solomon p. 101, in vertical form with the 16:15 ratio assigned to the larger numbers 120 and 112 ½.

(*medesimo semitono)* as in the diatonic.[81] This places the 16:15 semitone clearly as the third step with a smaller interval between it and the trihemitone. Although Ptolemy disapproved of the smallest interval in the middle position as not being "emmelic" (which was was reported by Gaffurius, as a criticism of Didymus), this is also what Vicentino did.[82] There is a clear statement of the use of major semitone (16:15) followed by the minor semitone (25:24) in the chromatic genus to confirm this.[83] These two semitones create the minor tone (10:9).

The interval in Vicentino's enharmonic lute tetrachord adjacent to the 16:15 semitone is however not 25:24 (70.67 cents), but 47:45 (75.28 cents), which we have seen (in Table 3) is a consequence of the arithmetical division of the 10:9 minor tone into 5 parts together with its placement adjacent to the 16:15 semitone.

Vicentino's trihemitone is intended to be a just intonation minor third (6:5), as was also used by Ptolemy, Eratosthenes, and Aristoxenus in the chromatic genus.[84] However, Vicentino's lute division of the minor tone into 5 parts creates a junction at 18.8" whereas for the 6:5 interval it should be at 18.75" (18¾"). The result is that the minor third is actually 311.03 cents, not the just intonation 315.64 cents.

It might present a surprise that a tetrachord created with whole-number ratios does not yield an exact minor third, but the consequences of Vicentino's lute division are clear: it is "optimised" for the diatonic and enharmonic; in the chromatic, slight inconsistencies have to be tolerated. Seen in another way, Greek tetrachords for all three genera will not necessarily yield a compatible tuning system when combined to a *single* division.

We can see from Table 4 that Vicentino's chromatic tetrachord, with the smaller second interval, is in the spirit of Didymus, not of Ptolemy. Wild has given an explanation of Vicentino's choice: "...none of the chromatic species of 4th, 5th, or 8ve begins with a minor semitone. This maximizes the contrast—always an important consideration for Vicentino—between the chromatic and enharmonic species: in the enharmonic species that begin with an enharmonic diesis, it is always a minor diesis, never a major diesis. Thus Vicentino's 4ths, 5ths, and 8ves as they appear in L'Antica musica allow the listener to distinguish the chromatic from the enharmonic genera as early as their very first interval." [85]

9. Further inconsistencies

Initially in this study, for simplicity, only two intervals were examined, which were described by Maniates as yielding inconsistencies, since the sizes did not fit the *Archicembalo*. Through the enharmonic lute division we have seen how Vicentino's

[81] Ibid., my translation of "...il terzo grado è il medisimo semitono del Diattonico."
[82] Gaffurius, 1518, Liber Secundus, Cap. 17, XXXIX [PDF 90], citing Ptolemy, [Book 2] 13. Ptolemy's criticism of Didymus' chromatic tetrachord is at [68.27], Solomon p. 96. Gaffurius states that Didymus erred (Miller p. 102) because in the enharmonic the lowest proportion should be the smallest. This is Gaffurius' extension of Ptolemy's idea, not Ptolemy's, who refers only to the diatonic and chromatic.
[83] Vicentino Book I, Cap. VII, fol. 14r°. Translation in Maniates, 1996, pp. 46-47.
[84] See Ptolemy [71.7], Solomon p. 101. Vicentino describes the minor third at Book I, Cap. VII, fol. 14r°.
[85] Wild, note 27.

system was constructed from elements of just intonation, yet even here there was no exact fit for the just intonation 6:5 interval.

There are four other intervals (given in Book V), larger than the "normal" ones and intended by Vicentino for giving particular effect in performance; these were also seen by Maniates as inconsistent. I have added the minor third to Maniates' list, and included the minor and major semitones for comparison.[86]

Table 6.

Vicentino Book V	page	interval ratio	interval size in cents	enharmonic lute cents	Rossi cents
minor semitone	143v°	21:20	84.47	84.47	77.5
major semitone	143v°	14:13	128.30	119.47 [15:14]	116.1
minor tone	144r°	13:12	138.57	144.35 - 165.00	154.8
major tone	144r°	8:7	231.17	238.89	232.3
minor third	144v°	6:5	315.64	311.03	309.7
proximate minor third	144v°	5½ : 4½ (11:9)	347.41	348.26	348.4
proximate major third	145r°	4½ : 3½ (9:7)	435.08	422.76	425.8

In order to show how these larger sizes could be represented on the *Archicembalo*, the tones Rossi gives for a 31-note, *equal* division of the octave have been retained from Maniates as a reference in the sixth column.

For the sake of completeness we can now examine the status of these additional ratios. Are they an inherent part of his lute division?

We have described enough of Vicentino's arithmetical division that it will be apparent his sizes are variable depending on their position in the tetrachord. Although the proximate minor third from the lute division (a minor third "enharmonically inflected", i.e. larger than a minor third by a diesis) appears to have a size close to Vicentino's 4½ : 3½ ratio, this is 9 steps from the *lowest* tone. If we were to suppose an interval occupying the upper 9 steps (dieses) of the tetrachord, the size would be only 336.88 cents, some 12 cents smaller.

For the same reason, the size of the "13:12" minor tone (nominally 4 dieses) depends on where the steps are located, which is why it is shown as varying in Table 6 from 144.35 to 165.00 cents.

We could perform any number of calculations which would show that some intervals Vicentino describes cannot fit the sizes on his division, let alone an *Archicembalo* with a tempered tuning. None of this would apparently have troubled him since he describes interval combinations such that the minor semitone equals the major diesis, even though in most cases *on his division* this will be incorrect.

[86] Maniates, 1975, Table 5, p. 344, which includes Rossi's sizes, but with my rounding of the cents. An extended list of intervals is presented also as Table 5 in Maniates, 1993, p. 35.

The issue is now less about whether the whole number intervals fit the *Archicembalo* tuning, but rather how Vicentino could have tolerated such large discrepancies even *within* the division he had constructed based on whole-number ratios, but hidden from us in the lute division.

We can better understand the issues involved if we examine another solution to essentially the same problem, as provided by Zarlino's 2/7 comma tuning and combined enharmonic system.

11. Zarlino's solution

How one can avoid the inconsistencies we have shown in Vicentino's tetrachords was demonstrated by Zarlino's remarkable system, which coordinated his version of the enharmonic with the diatonic and chromatic tunings. The starting point of his entire system was the 2/7 comma tuning, which he devised by distributing the syntonic comma: the fifths were flattened by -2/7 comma, but the size of the chromatic semitone was unaltered. This was shown with the range c-e¹ in a diagram, Fig. 9 below.[87]

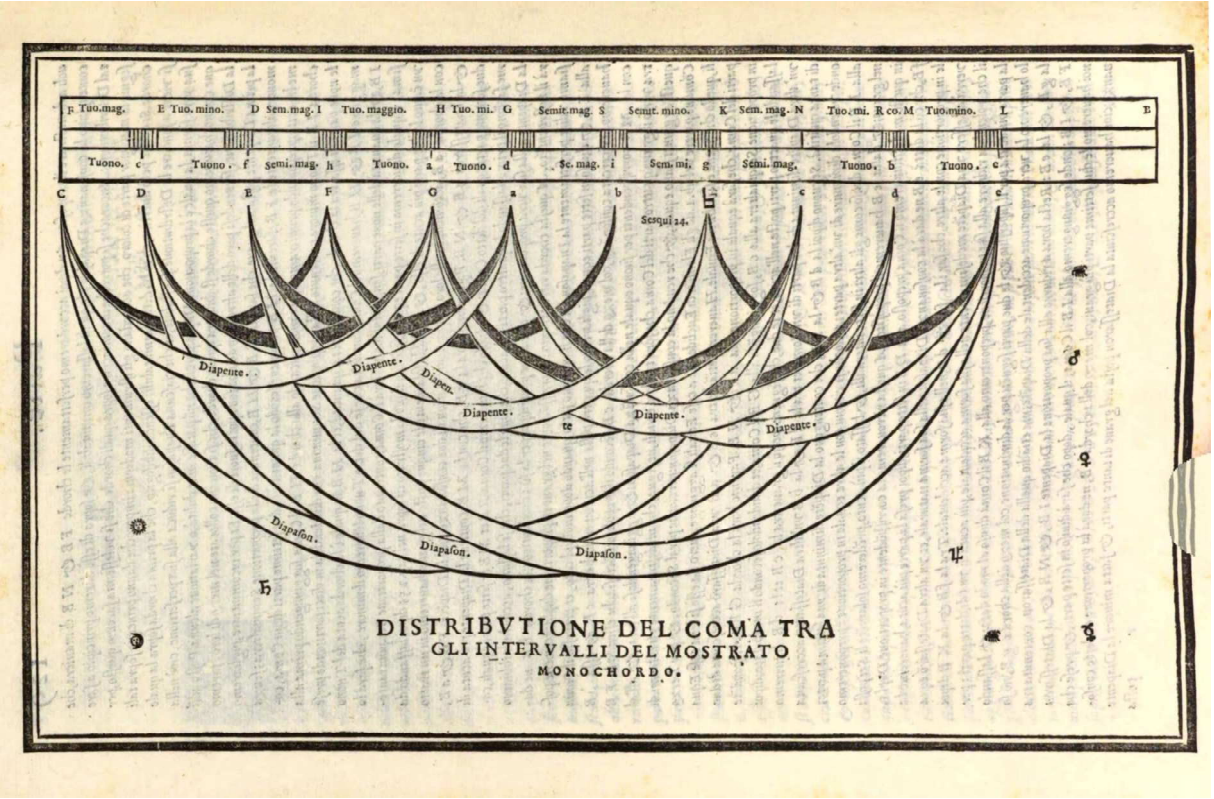

Fig. 9. Zarlino's 2/7 comma tuning system
Source: by permission of the Bayerische Staatsbibliothek

[87] Zarlino, 1558, p. 130 (the page is not numbered in this edition). Zarlino's constructional procedure is not easy to follow since some intervals are shown in proportion to their actual size, and there are effectively different scales within this diagram. For a description of Zarlino's procedure see Wraight, 2024, 'The tuning...'. The distribution of the syntonic comma is shown in Table 8. Perhaps surprisingly in such a complicated diagram, there are no mistakes.

Since Zarlino based his 2/7 comma tuning on the diatonico sintono, which essentially followed Fogliano's division, the choice of this tetrachord for his diatonic monochord is not surprising.[88] It is shown in the following way and called *sintono* or *incitato*:[89] The tetrachord B [♮ or B quadro] – E was presented vertically by Zarlino when citing the ancient sources, as in the Greek manuscripts.[90] Zarlino sometimes used the horizontal presentation with the lowest note at the left, as in any keyboard instrument (seen by the player), and this form is preferred here in order to make comparisons compatible with the usual monochord.[91]

Zarlino: diatonico sintono (or Incitato), 1558, p. 83 [PDF 101]													
♮ 48			C 45					♯ 40					E 36
16:15			9:8					10:9					

Zarlino's diatonic has the ratio 10:9 in the highest position, but the order of the whole tones is reversed in part of the later octave monochord presentation, described as "Tetrachordo Diatonico sintono di Tolomeo".[92]

The chromatic tetrachord Zarlino chooses is identical with that of Didymus, which we have already seen in Table 4. Zarlino does not give it a name, only that it is chromatic because it is very close to the soft chromatic of Ptolemy, but Galilei correctly identified its origin.[93] This is the natural choice because it fits the Fogliano division, and does so without the inelegance that Vicentino suffered with the ratio 47:45 at the second step (C). Zarlino's tetrachord is shown for B♮ – E below.

Zarlino: chromatico, 1558, p. 138 [PDF 156]													
♮ [B quadro] 80			C 75		♯ 72								E 60
16:15			25:24		6:5								

Zarlino's enharmonic tetrachord is shown below for E – a.

Zarlino: enharmonico, 1558, p.140 [PDF 158]													
E 400		x 384	F 375										a 300
25:24		128:125	5:4										

[88] Zarlino, 1558, Cap. 40, pp. 123-125.
[89] Zarlino, 1558, p. 83 [PDF 101], given in a vertical presentation with the longest "string" at the bottom.
[90] Solomon, p. 50 and 52, does not show any manuscript in facsimile, but he presents Ptolemy's tables of tetrachords vertically with the largest number at the lowest point.
[91] For example, Zarlino 1558, p. 122 for the diatonico sintono.
[92] Zarlino, 1558, p. 122.
[93] Zarlino, 1558, Cap. 46, p. 137. "...si può dire, che sia Chromatico: percioche molto si accosta al Chromatico molle di Tolomeo:". Galilei, Discorso, p.44, describes it as taken from Didymus, arguing that it is even older than Ptolemy. Consulting the line of transmission in Table 4 shows that Galilei is correct.

Although Zarlino had earlier described Ptolemy's enharmonic he avoided mentioning that his own enharmonic differed from that of the venerable authority, drawing attention only to the point of agreement in the consonant 5:4 ratio.[94] The tetrachord Zarlino gave us as "*quel di Tolomeo*" is shown below.[95] We may surmise that Zarlino, like Vicentino, probably learned this from Gaffurius' publication since Zarlino even used the same number examples, i.e. 368 - 276.[96]

"Enharmonico di Tolomeo", 1558, p. 85 [PDF 103]													
368	360		345										276
46:45	24:23		5:4										

We can now align all Zarlino's tetrachords in oder to show more clearly how they fit together.

Zarlino: diatonico sintono, 1558, p. 83 [PDF 101]													
♮ 48			C 45					♯ 40					E 36
16:15			9:8					10:9					
			5:4										

Zarlino: chromatico, 1558, p. 138 [PDF 156]													
♮			C		♯								E
16:15			25:24		6:5								
			5:4										

Zarlino: enharmonico, 1558, p.140 [PDF 158]													
E 400		x 384	F 375										a 300
25:24		128:125	5:4										
16:15													

Now that Zarlino's system has been described it becomes apparent that Vicentino's presentation is not that of a Ptolemaic tetrachord with the smallest interval at the lowest note, i.e. at B or E. Instead he has given us a monochord form, based on "string lengths" where the shorter "strings" are at the right.

[94] Zarlino, 1558, p. 139 [PDF 157]: "Il Ditono adunque che pone Tolomeo nel suo Tetrachordo Enharmonico, posto nel cap. 37 sarà al nostro proposito: percioche è intervallo consonante..." The "dissonant" interval with which Ptolemy's Ditono was compared was the Pythagorean 81:64 ratio; see Cap 37, p.117 [PDF 135]: "...il Ditono di proportione Super 17. partiente 64 [81:64] che è veramente diſſonante. Et perche forse alcuno potrebbe credere, che quella specie di Enharmonico, che ritrouò Tolomeo, facesse l'harmonia perfetta:"

[95] Zarlino, 1558, p.117, develops this tetrachord (of p. 85) to an octave monochord with the terminal values 38088 and 19044.

[96] Gaffurius, Liber Secundus, XLv° [PDF 93], Miller p. 105, and Table 4.

We have to imagine the mirror image of Vicentino's lute division in order that it can be compared with Zarlino's tetrachords. The *terminating* inch measurements of the enharmonic lute division have been added as an orientation, in the second line, their position now inverted.

Vicentino's enharmonic lute division in mirror image tetrachord form												
16⅞"												**22½ "**
46:45	24:23		5:4									

"Enharmonico di Tolomeo" according to Zarlino, 1558, p.117 [PDF 135]												
25392	24840		23805									19044
46:45	24:23		5:4									

Zarlino: enharmonico, 1558, p.140 [PDF 158]												
E 400		x 384	F 375									a 300
25:24		128:125	5:4									

Through this presentation it is easy to recognise that in the enharmonic tetrachords Vicentino and Zarlino use different interval sizes, and place them differently. Vicentino's tetrachord follows Ptolemy; Zarlino's clearly does not.

In the Ptolemy-Vicentino system the enharmonic starts with 46:45 ratio interval (38.05 cents), whereas according to Zarlino's system, the enharmonic genus *starts* with the step E-E♯ or B-B♯ even though this step (MD) is the *same size* as the chromatic semitone (mS = 70.67 cents). This lack of a clear distinction between the chromatic and enharmonic tetrachords is the infelicity Zarlino accepted in order to coordinate his enharmonic with his tuning system.[97]

When an enharmonic step is made in Vicentino's system, say from B, then the next step in the tuning is C♭. Vicentino places this note on the upper keyboard (row 4) and calls it B4°, which is logical in his system since it is the "enharmonic inflection" of B.[98] Thus, the enharmonic genus as Vicentino conceived it is not merely different from Zarlino's; it is not even playable on the 24-note harpsichord Dominico Pesarese built for him in 1548: there is no C♭.[99]

[97] These enharmonic keys might have been given the colour red in the instrument which Dominico Pesarese built for Zarlino in 1548, as Rasch has suggested, p. 46. It is not certain whether the red colour can be attributed to the enharmonic keys of Zarlino's instrument since he only mentions the drawing, on which we *should* see red keys: Zarlino 1558, p. 140, under the diagram, " quanto vaglia l'Arte aiutata dalla Natura, nel congiungere, & collocare mirabilmente, con bello, & regolato ordine le chorde Chromatiche tra le Diatoniche; & tra l'vne, & l'altre di queste, le Enharmoniche; Le quali si conosceranno nel Tastame delli detti Istrumenti in questo: che a differenza delle diatoniche, & delle chromatiche, si porranno di colore rosso; come nel sotto posto istrumento si può vedere." It appears that the printer didn't accede to the extravagance of printing red on the keys, or Zarlino couldn't afford it; this specification was dropped in the 1573 edition, where the text reads: "si porranno d'un altro colore", but the following "come nel sottoposto istrumento si può vedere" was left standing, is now redundant and makes no sense.

[98] Vicentino, see the fold-out drawings of the keyboards, usually at the end of the book.

[99] A detailed discussion of Zarlino's 24-note harpsichord and its limitations is found in Wraight, 2024, 'The tuning...'

Zarlino's tetrachord system was designed to fit the Fogliano monochord, thereby producing a workable division of the octave, which Lindley has described as "mathematically correct" and "quantitatively exact".[100] Vicentino might have adapted the Fogliano monochord to fit Ptolemy's enharmonic, but he ignored the problem of the octave division, or was incapable of analysing it.

Whereas the sizes of intervals Zarlino uses in his enharmonic tetrachord are not to be found in any known Greek sources, Vicentino's system mostly follows Ptolemy, so he can fairly be credited with a revival of Greek music.[101]

12. Evaluation

We have moved the discussion from wondering why there should be a "glaring discrepancy" between Vicentino's 21:20 ratio (84.5 cents) and the actual size of a semitone in the *Archicembalo* (c. 76-77 cents) to another area. The 21:20 interval was not the result of a clumsy attempt to find a ratio for the *Archicembalo's* minor semitone, but a calculated and intended part of his enharmonic lute monochord which divided the tone (arithmetically) into 5 parts. Thus, the question must now be why Vicentino thought it unproblematic that this ratio be so far removed from that which was used in his *Archicembalo*.

Furthermore, when we reflect that Vicentino describes the two-diesis minor semitone (mS) as equivalent to the major diesis (MD), even though the mS can vary from 84.47 to 75.28 cents on his division, then we realise that despite the results in his enharmonic lute division he considers all the dieses (of the same type) to be of equal size. The contradiction could not be clearer.

Although we often work nowadays with cents, which allow us to ascertain quickly the relative size of intervals, we should not suppose that in Vicentino's time, or even earlier, small size differences went undetected.[102] Boethius, repeating his Greek forebears, clearly states that a tone cannot be divided into two *equal* parts by arithmetical methods, and Vicentino was aware of this as well.[103] Part of the explanation for the use of the arithmetical method is that there was no *numerical* alternative, until logarithm tables became available.

Could Vicentino instead have published a drawing with correct, geometrically-derived equal divisions? He could have, but essential for 16th-century theoreticians was the use of *number*. As Barbera observed of Aristoxenus, Eratosthenes, and Ptolemy, "they are all Pythagoreans or Neo-Pythagoreans firmly entrenched in the ideal and

[100] Lindley, 1997, p.116.
[101] I have consulted Chalmer's list of enharmonic tetrachords, pp. 167-172, to produce this assessment.
[102] Fogliano IXv° illustrates the principle in textbook fashion of cross multiplying ratios in order to extract the difference. See also Gaffurius 1518, Liber Primus, XXIr°-XXIIv°, for the same type of calculation.
[103] Boethius, Book III, 1 fols. 268-269. Boethius restates this following the analysis of the genera at Book IV, 7, fol. 323. Vicentino discusses this, Libro della theorica, Cap XVI, fol. 6v°. See Maniates, 1996, translation pp. 18-20.

obsessed with the power of numerical representation of the physical world – or, more severely, they see the physical world as a material representation of number".[104]

Zarlino's *senario* of the ratios formed from the numbers 1 to 6 was also an expression of this and links him with the Pythagoreans that Barbera mentioned.[105] Furthermore Zarlino's delineation of music theory not only reported the alignment of the planets with the ratios of harmony according to the Ancients, but as Celhoffer has remarked also showed the astrological symbols for the seven planets together with the diagram of the 2/7 comma meantone tuning.[106]

The numbers in traditional music theory formed superparticular ratios: 2:3, 3:4, 4:5, and 5:6. A modern definition of a superparticular ratio, such as "a ratio in which the antecedent exceeds the consequent by 1" does not help in understanding some of the attraction of this type of relationship.[107] Nichomachus gave another explanation which hints at the cosmological position of number when he defined the larger number in a superparticular ratio as "...a number that contains within itself the whole of the number compared with it, and some one factor of it besides".[108] Here we have the idea of the *connection* of a number to a larger scheme of numbers. Position and relationship are thus the essential ideas for all that exists.

A further fascination for the Ancients, and apparently Renaissance scholars as well, were *series* of superparticular ratios. When we consider the series 2:3, 3:4, 4:5, 5:6, 6:7, 7:8, 8:9, 9:10, only 6:7 and 7:8 had less significance for Renaissance music practice. There was a virtually seamless continuum of superparticular ratios describing the musical firmament, or even comprising the Ideal World.[109]

Vicentino's enharmonic lute division comprises three overlapping series of superparticular ratios which gives it its remarkable character and elegance: 45 to 40, 50 to 45, and 48 to 45 for the major tone, minor tone, and semitone respectively. We can also recall Ptolemy's equal diatonic genus which comprises the superparticular series 10:9, 11:10 and 12:11.

Although such arithmetical division of intervals into smaller parts as Vicentino practised cannot yield equal sizes of intervals, the sequence of superparticular ratios has an undeniable place in the history of music and had its attraction for reasons which have been outlined above.

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[104] Barbera, p. 314. It should not be supposed that this problem has become obsolete: Penrose (before he became a Nobel laureate) recently took up this theme again, the question being what sort of number is adequate for description of the physical universe. The enquiry resulted in a very substantial book.

[105] Zarlino, 1558, pp. 27-28. Palisca, pp. 247-249, gives a summary of Zarlino's *senario* theory.

[106] Zarlino, 1558: pp. 101-103 on the Ancients and their planetary alignments with music, p. 130 on the diagram of the 2/7 comma tuning [the page number is not printed in the 1558 edition]. The observation about the planets and cosmology in the 2/7 comma tuning is due to Celhoffer, pp. 54-55. For some unknown reason the planets were removed from the diagram in the 1573 (second) edition.

[107] This definition is given by Barbour, xii.

[108] Barbera, p. 320, note 5, gives this definition. See Nichomachus Book 1, Chapter XIX, in D'Ooge, p. 215.

[109] This also overlapped into practical work. When stringing a harpsichord, Cristofori's use of 10 strings with gauge 10, 9 strings with gauge 9, 8 strings with gauge 8 (etc.) is an interesting echo of the theoreticians' work. See Wraight, 2000, on Cristofori's "numerical progression", esp. pp. 176-181.

Vicentino gives us an inkling of his own thinking on these matters when he declares, after tediously listing all the topics in Boethius which he did *not* mention, including the impossibility of equal division of a tone, that: "I passed over all these topics because none of them has any value whatsoever for our practice."[110] Thus, it seems that the dichotomy we are dealing with is that of *musica speculativa* and *musica practica*.

Learned disputes on this distinction between the *speculativa* and *pratica* were still being conducted when Vicentino was a youth. Ramos had published a tuning in 1482 in which a major third with the ratio 5:4 was proposed instead of the Pythagorean ratio 81:64 (composed of two 9:8 tones) and thereby attracted criticism, among others, from Gaffurio. Spataro leapt to his teacher's (Ramos') defence in 1521 explaining that only in musical practice was 5:4 correct; in speculative music the ratio was still 81:64.[111]

Even if tables of logarithms had been available 100 years earlier for Vicentino's use, he would probably not have calculated with them since the tetrachords of Ancient music were based on whole number ratios. He could only work with these, and somehow ignore the discrepancies which inevitably arise between whole-number divisions and keyboard temperaments.

It seems as if we would be justified in detecting some guile in his whole procedure, and the concealment of the structure of the lute division; there is even the mention of "*molti segreti*" on the title page. Although Book V with the enharmonic lute now appears at the end of the volume, Maniates observed that Book V could have stood alone.[112] Thus, Vicentino, raher than keep it a secret, might have intended to discuss the enharmonic lute division more fully, but a full elucidation was never worked out.

That Vicentino naïvely believed his enharmonic lute division could be used in a practical instrument is hardly credible, especially now that its design has been revealed. The difficulty in interpreting the evidence is that Vicentino oscillates between the poles of acknowledging the inconsistencies and ignoring the difficulties.

Perhaps the explanation is that Vicentino's thinking lacked rigour and system, but also that the competance was missing, which Zarlino possessed, and used to construct his 2/7 comma tuning by division and distribution of the syntonic comma. This does not denigrate Vicentino, but rather elevates Zarlino's own achievement. The temperament Vicentino needed was not quantifiable in his time; it required Rossi's work some 100 years later to provide it.

If we see the objective of *L'Antica musica* as describing a practicable tuning for the *Archicembalo* then clearly Vicentino failed. Zarlino succeeded in providing a mathematically correct tuning for his *clavocembalo*, but his enharmonic genus had to fit the Procrustean bed of his tuning system, and he composed nothing for it. Vicentino's achievement was to recreate the enharmonic genus and give it a clear musical purpose.

Vicentino's perception of this problem appears to be a purely practical one: that the small difference between the major and minor tone (a syntonic comma, 21.51 cents)

[110] Maniates' translation, 1996, p. 20.
[111] See the discussion in Fose p. 58 et seq.
[112] Maniates, 1996, xxxvii.

cannot be heard in singing and playing. Of course the reason why this small difference would not be heard is because it would not be made by competent musicians playing variable-intonation instruments (e.g. violins); they would play well-intoned intervals, or whatever they took to be "well-intoned". Only with the tuning of a fixed-pitch, e.g. keyboard, instrument would one arrive at a "recognition of the difference".[113] This appears to be his own answer to the question.

This is a crucial admission by Vicentino since it allows us to infer that he understood that his *keyboard* tuning could not be constructed with unequal sizes of whole tones. In designating semitone sizes in the *Archicembalo* from his tetrachord system based on unequal whole tones he took a step which would lead, on his own evidence, to audible discrepancies. This much must be clear, that he recognised his tetrachord system and the *Archicembalo's* temperament to be substantially different; the enharmonic lute tuning was perhaps the *musica speculativa*, but ultimately only the *musica prattica* and his *moderna prattica* mattered.

[113] My text paraphrases Vicentino's statement Cap. LXI, 143v: "...questa poca differenza di uno, & dell'altro non si può sentire cantando ne sonando, ma nello accordare li strumenti si perviene alla cognitione de si poca differenze..."

Appendix

The construction and dating of Vicentino's Archicembalo

That this enharmonic lute division and the *Archicembalo* were designed with the Roman foot, as was argued above, suggests the involvement of a skillful instrument builder who was able to combine practical measurements for string lengths with the theoretical basis of the division of the tetrachord. Alternatively, we would have to assume that Vicentino was acquainted with such details of instrument making, which seems prima facie unlikely. The details of construction described below argue for the activity of a professional, but as yet unknown instrument maker.

Vicentino's design for the *Archicembalo* is given by printed lines at the beginning of Book V, the length of which is to be used, or multiplied by a certain amount. The "first string of the soprano" is 158 mm and is to be taken two times, i.e. 316 mm.[114] As Tiella correctly argued, this cannot be the top string, c^3, since it would be far too long.[115] It is not clear where Vicentino understood the "soprano" to commence since his own definition of the soprano voice gives a range of c^1-f^2.[116] Tiella's plan, interpreting Vicentino's instructions, shows that a c^2 = 316 mm leads to a feasible design.[117] Tiella's C string length of about 1700 mm is a convincing dimension which can be fitted into a case length of 1950 mm (20x 97.5mm, the length of the line given), excluding the keyboards.

When a harpsichord has a c^2 = about 280 mm it is usual for the entire case length (including keys) to be about 1950 mm and the C length to be about 1500 mm, thus Vicentino's design does indeed appear to be a slightly larger (lower-pitched) version. Although built well over 100 years later in Rome, the 1656 harpsichord by Zenti would represent the size of instrument that Vicentino had in mind for singing.[118] Thus, Vicentino's instructions are plausible and consistent with what we know of Roman harpsichord making.

Is it possible to find any indications in the *Archicembalo's* design which would corroborate the pitch indication and further support the idea it was constructed in Rome?

The drawing shows the wrestplank with some strings, jacks and a nut position. That the nut position is correct in the treble is confirmed by Vicentino's dimension for the distance from nut to jack.[119] Although no bridge is shown, the length between the middle row of jacks and the nut is consistent with Roman harpsichord building practice, both in the treble and bass. Thus, we have a plausible size for the first string in the treble, if it is taken at actual size.

Further analysis of the keyboard drawings shows that some dimensions can be credibly expressed in Roman inches.

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[114] Vicentino, Book V, Cap. II, fol. 100v°: "Linea che va due volte longha per la lunghezza della prima corda de i soprani."
[115] Tiella, 1975, p. 141.
[116] Vicentino, Book IV, Cap. XVII, fol. 80: "Essempio de i Termini del soprano"
[117] Tiella, 1975, p. 135. This case length does not include the keys.
[118] Musée Instrumental de Bruxelles, no. 1600. Wraight, 1997, Part 2, p. 318.
[119] The first line on fol. 101r° = 61 mm, which agrees with the drawing. Brink, p. 81, believed the nut position to be incorrect.

The drawings, usually bound at the back of the book, were produced on large sheets of paper, the printed area being about 28 cm by 36 cm. Two of these were intended to be joined at the short side for the wrestplank with strings and jacks drawing. The keyboards were composed of three sheets joined at their long sides. Since the Lowinsky facsimile edition shows complete drawings, not the separate sheets, this detail has not been obvious to casual students of the book.[120] The Cremona, Rome and Munich copies all contain original sheets in their un-joined condition. There is at least one copy in which the drawings have been assembled (Sibley), but others in which the drawings are partially (Munich) or completely absent in the digitised copies (Vienna, Zürich, Paris).[121]

Assembling the sheets to make a complete drawing is not easy if working from the digital copies because the keyboard sheets were folded by the printer to fit the book size and were often not unfolded for scanning. However, the individual sheets contain some errors (especially the wrestplank drawing) so that the completed drawings require interpretation.

Brink gives dimensions of the Sibley copy which he studied, but even these do not agree with some other copies.[122] The Cremona, Rome and Munich digital copies agree in the size of the keyboards, as far as the horizontal dimensions across the keys are concerned, so their evidence has been preferred to the Sibley assembled-drawing.[123] The digital copies yield a dimension of 647 mm across the keycovers = 26" Roman inches (1" = 24.825 mm). It has been found in an empirical study of Italian harpsichords that the width of the keyboard was often expressed in round numbers of the local inch measurement, as is the case with Vicentino's design.[124] The width of the keyboard could remain unchanged, regardless of the different pitches.

However, the width across the *keyblocks* = 735 mm = 29½" Roman inches. If we were to include the usual thickness of the case sides then the overall outside case width is 30".[125] It has been found that the harpsichord design usually proceeded from the keyboard outwards, with the result that the *overall* width was not necessarily a round number of inches. In Vicentino's design the ratio of width to length is 5:8 so attention was clearly paid to proportion. However, the length Vicentino gives (20x 97.5 mm = 780 mm) does not yield the width of the keyboards and wrestplank he has

[120] The keyboard drawings are obviously reduced in size in Lowinsky's facsimile edition since they do not even agree in their width. The wrestplank drawing is reduced by 14 mm compared with the digital copies or 16.8 mm compared with the Sibley copy, a detail which Brink missed; he described it, p. 31, as the same size as the Sibley copy. The Sibley wrestplank drawing is possibly incorrectly assembled since Brink, p. 67, gives 737.8 mm as the width, which was 6.1 mm wider than the Sibley keyboards. This discrepancy may be due to faults in the printing blocks themselves, a subject which will not be examined further here. Brink's discussion of the problem with the jackslides is at p. 69 et seq.

[121] Maniates, 1996, pp. 318-320 reproduces fully asembled drawings credited to the British Library's Hirsch 1 copy, but all the drawings are actually from Lowinsky's facsimile. To add to the confusion, some digital copies contain scans of the Lowinsky edition *drawings*, but a different title page (e.g. Library of Congress).

[122] Brink, Figs. 8-10: both keyboards = 731.7 mm. Wrestplank = 737.8 mm (p. 67).

[123] One has to select the better scans from each source. Cremona has the flattest drawings, but the treble end of the Rome lower manual has less distortion at the keyblock. The Rome copy scan lacks the edges of the wrestplank and in the Munich copy the wrestplank drawing is missing.

[124] Wraight, 2011.

[125] The keyhead division is commendably even in both keyboards. No allowance has been made here for paper shrinkage since the printing.

drawn. The difference of 45 mm between 780 mm and 735 mm is too large to be explained away by slight inaccuracy in the drawings or printing process; it represents an error of 5.6 mm on the page, if we take the actual width of the keyboards to be correct.

The natural keycovers are 2½" long, i.e. the two keyboards' keycovers are 5" long. The slope of the jackslides is given by 15" in the bass and 12" in the treble, at the sides of the wrestplank. This assumes that the dimension indicates the position of the bellyrail, some ½" behind the third jackslide.

Thus, the *Archicembalo* design is consistent with an origin in Rome, using Roman inches.

The association with Rome sheds light on another issue because it was previously unknown whether Vicentino's instrument was made in Ferrara or Rome. Since Vicentino was resident in Rome by the summer of 1549, with his employer Cardinal Ippolito d'Este, it would appear that the *Archicembalo* described in the book was made between 1549 and 1555 in Rome.[126]

It is strange that Vicentino should have given dimensions for a harpsichord that was not suited to a pitch for singing, since his book is about vocal composition.

Given this inference that the Roman *Archicembalo* was built between 1549 and 1555, Zarlino's enharmonic *clavocembalo*, built in 1548 was slightly earlier, albeit with a smaller enharmonic range since it only had 24 keys in the octave, lacking the double flat accidentals.[127] Salinas made a claim for a yet earlier instrument, imprecisely dated to 1538-1540 or 1547, with a 24-note compass which was based on just intonation, incorporating split D keys.[128]

[126] See the discussion in Wraight, 2002, pp.115-120, of the various references to the manufacture of instruments. There may have been two *Archicembali*: one made in Rome and one left in Ferrara after Vicentino's departure from the Este court. The Roman *Archicembalo* could even have been the second instrument.
[127] See Stembridge, 1993, pp. 45-50.
[128] See Barbieri, 1983 for the dating and Wraight, 2002, p. 110, for a layout of the compass and the dating to 1547.

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