The tuning of Trasuntino's "Clavemusicum Omnitonum" and Zarlino's enharmonic system

Denzil Wraight www.denzilwraight.com/Clavemusicum.pdf

Abstract

The monochord made in Vito Trasuntino's workshop for his 1606 "Clavemusicum Omnitonum" with 31 keys per octave was designed with Zarlino's 2/7 comma tuning. Zarlino's diagram (*Le Istitutioni Harmoniche*, 1558) was the basis, but the whole tones were reproduced without tempering. Zarlino's 24-note/octave harpsichord (built in 1548) had a harmonic range G♭ - A♯♯, although Zarlino's design required a further eight notes. This range avoided 30 cent mistunings of tetrachords resulting from Zarlino's 2/7 comma tuning. Trasuntino's 1601 enharmonic harpsichord exceeded the G♭ - A♯♯ range, but the monochord reveals that Trasuntino changed the tuning system to avoid Zarlino's problem. The discrepant ♭♭ notes in the monochord were constructed with a 16:15 interval from the ♯♯ ones, leading to ♭♭ notes which were too flat in 2/7 comma. They would have been nearly correct for Trasuntino's new tuning: 31-note equal temperament [ETS 31], which was required for correctly-tuned tetrachords, and is close to 1/4 comma meantone. The Clavemusicum Omnitonum's monochord could not set the tuning because of the conflict of two tuning systems, but also from inaccurate manufacture. Nevertheless, the monochord documents Trasuntino's enharmonic instruments, which initially followed Zarlino's tradition, and differed from Vicentino's enharmonic system and his *Archicembalo*.

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Introduction

The 31-note per octave harpsichord made by Vito Trasuntino[1] (in 1606, Venice) for Camillo Gonzaga is the only instrument of its type known to have survived.[2] The nameboard describes it as a

CLAVEMVSICVM OMNITONVM
MODVLIS DIATONICIS, CROMATICIS, ET ENARMONICIS

thereby declaring that it could sound the three Greek genera:

DIATONIC, CHROMATIC, ENHARMONIC

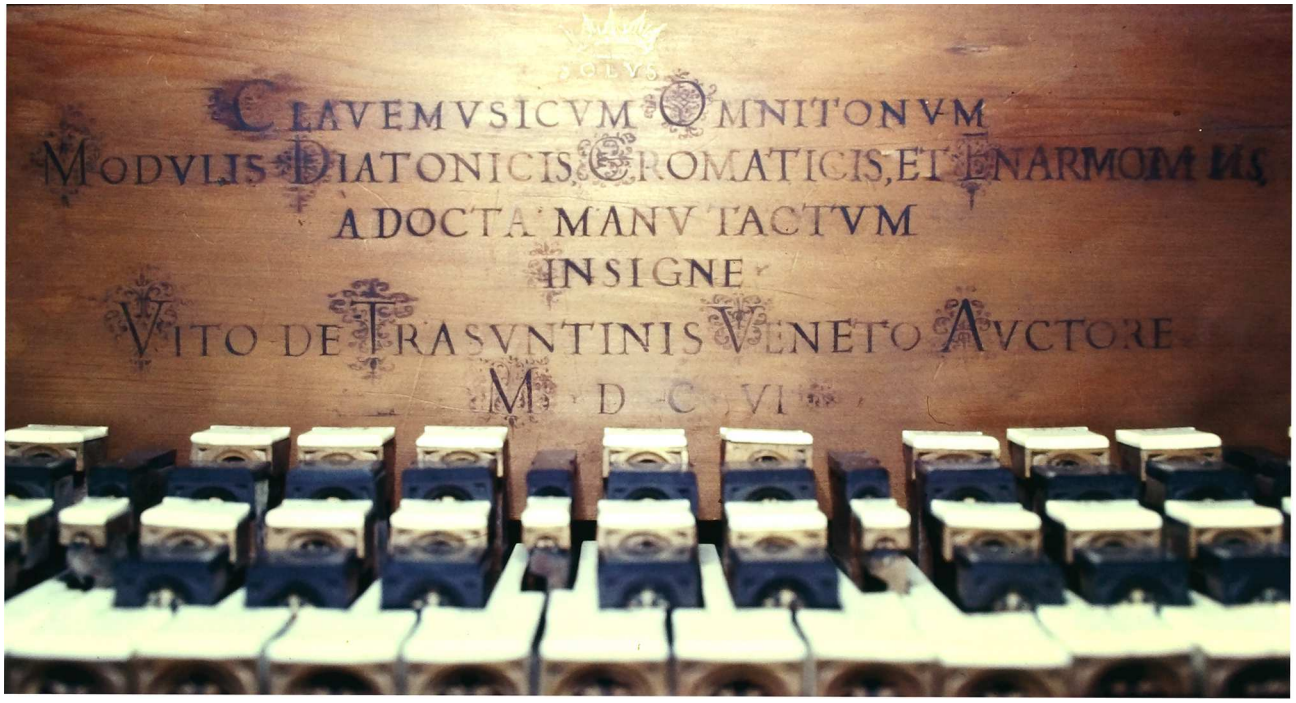

Photo. 1. The inscription on the nameboard and ranks of keys
Source: the author

Instead of the usual, single accidental note there are four, but also with two notes between E and F, and between B and C, as visible in Photo. 1. (This is clearer in photo 2, nos. 43 and 44, then 61 and 62).

[1] "Trasontini" is the version which occurs most often in official documents: See Di Pasquale 2019/1. Vito signed his name "Trasontini" in the letter to Camillo Gonzaga. See Appendix, although an ink smudge leaves a slight doubt about the last "i"; this is also the form used in his will. It might be more accurate to write "TrasOntini", but "TrasUntinO", the main version of the Dizionario Biografico degli Italiani, has been retained here in order to facilitate searches on this spelling. Di Pasquale 2019/1 (and private communication) preferred this version.

[2] Museo Civico Medieval, Bologna, inv. no. 1767, for the monochord, inv. no.1766 for the harpsichord. See Van der Meer, 1993, pp. 91-92 and pp.146-148 respectively. The word "ENARMONICIS" has been poorly re-written at some stage.

Photo. 2. Photo of an octave of the keyboard
Source: the author

How it should be tuned would appear to be clear since an "analogue tuning machine" was supplied, a monochord, with a range corresponding to c^1- e^2 of the harpsichord, and numbers 32-73 on both the monochord and keyboard to identify the notes. Bottrigari described the terror which such a complicated instrument as Vicentino's *Archicembalo* might occasion in even an experienced organist, so this tuning device appears to answer the practical difficulty of setting the tuning.[3]

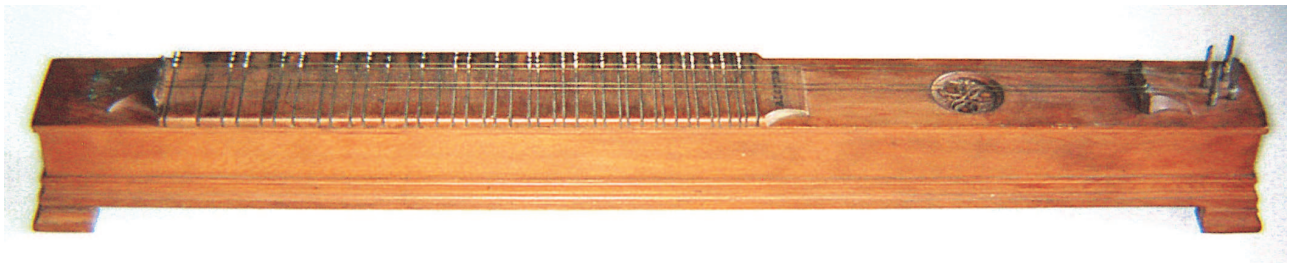

Photo. 3. Photo of the monochord
Source: the author

The ivory-ebony colour scheme of the small insets in the frets of the monochord does not match that of the keyboard, an infelicity which could cause a confusion that the monochord was supposed to dispel: on the keyboard the natural notes are ivory, but black on the monochord. The *normal* sharp and flat notes are black on both monochord and keyboard,

[3] Bottrigari, p. 41.

The monochord was so constructed that small bridges, equivalent to the frets of a viol or lute, *could* be inserted into the slots, from below (with the orientation seen in Photo 4), or from the left, as seen when the key numbers are at the right hand side. This would permit any combination of two tones from the two pairs of strings. In principle, four separate tones of a complete tetrachord could be sounded after inserting four separate bridges. The very name inscribed on the underside of the monochord "TRECTA CORDO" [sic. *recte* Tetra Cordo] could appear to imply this possibility.[4] However, the close spacing of the string pairs would make plucking the strings more difficult than would a wider and even spacing.

Photo. 4. Source: Museo Internazionale e Biblioteca della Musica [5]
The lowest note c¹ (no. 32) is at the left end, the highest, e² (no. 73) is at the right end.

Friedemann Hellwig, in 1981 at the Germanisches National Museum, Nuremberg, was entrusted with the conservation of the harpsichord, but he also examined and documented the monochord on the museum drawing.[6]

Hellwig established with a comparison of mouldings from the harpsichord and monochord that the tuning device must have been made in the Trasuntino workshop with their moulding tools.[7]

Marco Tiella found an inconsistent division in the monochord, which was "presumably inconsistent with meantone tuning".[8] In 1985 Hellwig wrote of the tuning: "An analysis of this tuning device shows something approximating a meantone system, a type of tuning with pure thirds but narrow fifths..."[9] A tuning with *pure* major thirds implies 1/4 comma meantone, which is how the tuning of the instrument is currently reported in wikipedia.[10] Despite these analytical results, in 2008 Patrizio Barbieri still considered the tuning of the monochord to be hypothetical.[11]

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[4] See Van der Meer, 1993, p. 91, although it is not stated whether this inscription, a misspelling for "tetra cordo" could have the same origin as the other markings on the normally visible (upper) side of the monochord. Here we find "VT" standing for Vito Trasuntino and "Al Unisono", signifying that all strings should be tuned to the same pitch. No such hypothetical bridges fitting the slots have survived.
[5] https://bbcc.ibc.regione.emilia-romagna.it/pater/data/bologna/bo006/bo006_03/049a.jpg (accessed 11.09.2021). The colour balance has been altered to match the other photos.
[6] The drawing is available from the Germanisches Nationalmuseum, Nuremberg.
[7] Hellwig, 1985/3, pp. 26-28: p. 26: "...and e) of the Clavemusicum" should read "...and d) of the..."
[8] Tiella, 1975, p. 141.
[9] Hellwig, 1985/2, p. 439: "Eine Analyse dieses Stimmgerätes ergibt angenähert ein mitteltöniges System, also eine Stimmungsart mit reinen Terzen aber verminderten Quinten..." Van der Meer 1987, p. 18, was less cautious and described the tuning as "meantone".
[10] The Wikipedia article "Archicembalo" describes the tuning as "with the usual meantone temperament for the first row of upper keys with C♯, E♭, F♯, G♯ and B♭". https://en.wikipedia.org/wiki/Archicembalo (accessed 15.10.2020). This Wiki information did not cite a source for this opinion. Van der Meer, 1993, (see note 2), did not describe the actual tuning of the monochord beyond summarising some of the difficulties Hellwig 1985/2 had found in the monochord's fretting. The expression "tuning" is used in this article also to include a temperament, i.e. with tempered intervals.
[11] Barbieri, 2008, p. 27.

Some of the frets (with the 𝄫 notes) were found to form smaller intervals than would be expected in 1/4 comma meantone tuning, so that an incorrect sequence of smaller and larger fret sizes resulted.[12] This is illustrated in a drawing of part of the monochord as Fig. 1 (below), which corresponds to the orientation of photo 3 above.

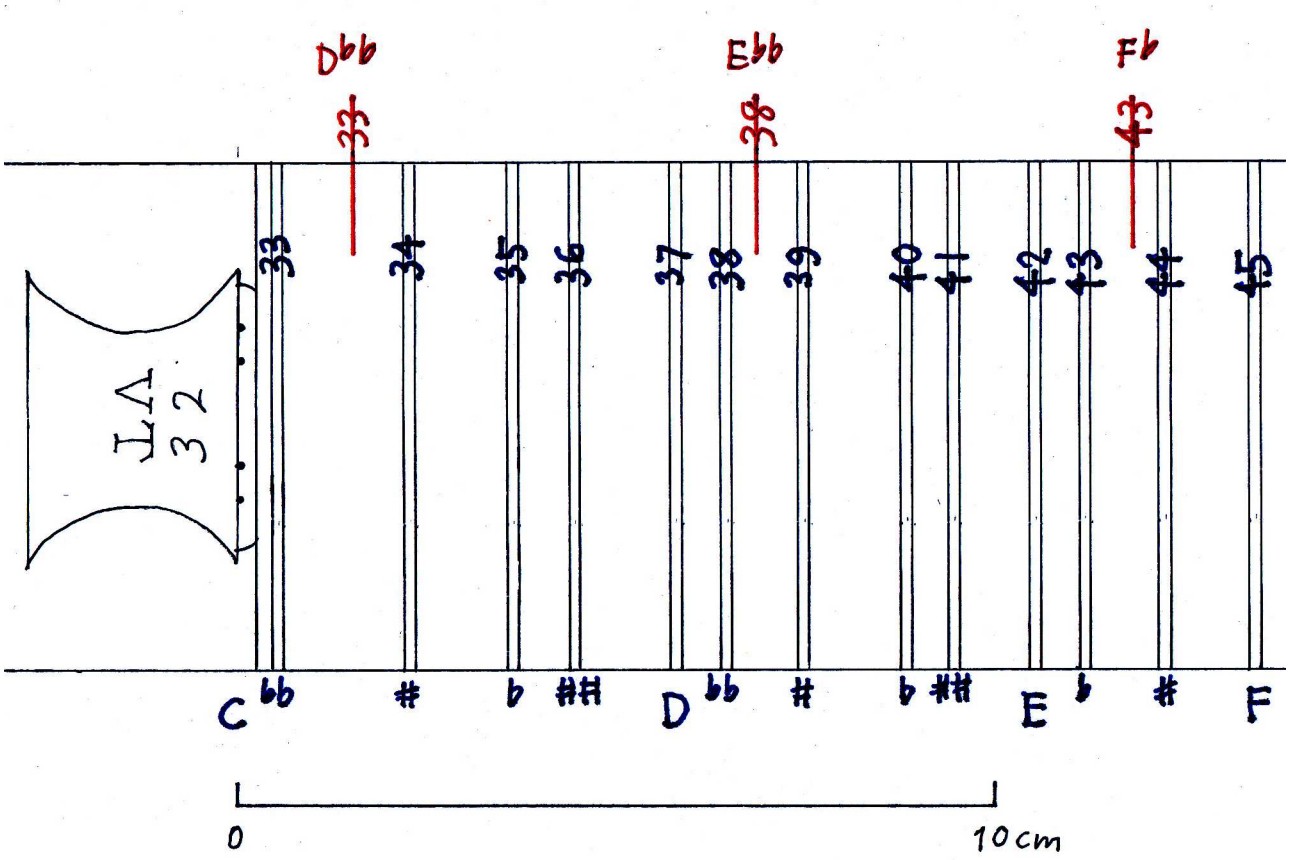


Fig. 1. The monochord's frets from C-F
Source: drawing by the author from Hellwig's 2015 data
Correct fret positions for 33, 38 and 43 are shown above in red

The names of the frets are given in blue (below the frets). The numbers of the frets are given over the fret slots, as on the monochord in Photo 4. The correct positions of D𝄫, E𝄫, and F♭ are drawn in red (above the frets). The divergence of D𝄫 (no. 33) from the correct position is considerable.

It is the aim of this study to examine

1. If the interpretation of the wikipedia 1/4 comma meantone tuning in the monochord can be confirmed or overturned.
2. To identify the tuning of the monochord.
3. To explain the idiosyncratic, 𝄫 frets, which are too flat.

[12] Klaus Martius, in an unpublished paper delivered at a conference held concerning the instrument (see Bibliography), suggested that the division of the notes from C-F in 1/4 comma meantone i.e. C - C♯ - D - E♭ - E – F, with these interval sizes in this sequence, formed the basis of the division of the whole tone into C - D𝄫 - C♯ - D♭ - C♯♯ - D, thereby explaining the incorrect placement of D𝄫 in this example, and all the small 𝄫 intervals.

1. Measurements of the monochord

Michael Thomas described the monochord briefly and gave measurements of the fret positions.[13] However, the measurements taken by Hellwig under better conditions have been preferred to Thomas' data, or the museum drawing; the data presented in Hellwig's 2015 paper forms the basis of this study (see Table 1).[14]

Hellwig did not state how the measurements were taken nor which points on the monochord were measured. According as the crown of the nut, the middle of the nut pin, the edge or middle of the fret is measured, measurements would vary up to about 1mm. These could even account for the slight mismatch of the c length (0.45 mm or 2.9 cents) compared with the octave below (C). Even if Hellwig's measurements describe the theoretically-correct vibrating length of a string, i.e. between the centre of the nut pin and centre of the bridge in the fret, the question would have to be answered whether the maker of the monochord used such dimensions in constructing the fretting. However, such considerations are secondary to the task in this article of examining the tuning system and will not markedly affect the conclusions drawn here.

2. A visual approach to determining the monochord's tuning system

My more recent approach to determine the tuning of any monochord, during the examination of clavichord fretting, used a visual comparison of theoretically-correct fret positions on transparent foil laid over a drawing of the actual monochord fretting. This is a method which enables one to test the accuracy of fret construction at a glance. Its advantage, compared with a numerical approach, is that one can quickly find a good fit, but also see *where* error occurs, which can be in part of an octave, and/or bridge positions. Furthermore, one has a practical grasp of the magnitude of error since in such examinations one has to bear in mind the practicalities of making an instrument, where errors of 1 mm can easily occur. Such errors are hard to assess in cents, which are somewhat non-intuitive: 6 cents, the narrowing of a fifth, may seem to describe a large difference, but it is equivalent to only 1.4 mm at the F on this monochord (no. 45).

How much manufacturing error one should expect in such a monochord is *a priori* unknown, but previous experience of examining Italian clavichord frettings suggests that makers could mostly restrict errors to within 1 mm. Applied to the monochord this represents an error of 6.5 cents at c (fret number 63), but half this an octave lower at C.

This visual method was used to test whether Zarlino's 2/7 comma meantone tuning might explain the construction of the monochord. In his illustration of this tuning, Zarlino showed a monochord division (see Table 4) with the range C - e, exactly as in Trasuntino's monochord, a coincidence which in itself is suggestive.[15] Thus, any intelligent worker should have been able to construct Zarlino's tuning system on a real monochord, although

[13] Thomas, p. 149-151. The measurements differ from those given by Hellwig, 2015, by typically 1-2 mm, but up to 3mm (rounded values). The monochord is also mentioned briefly in Tiella 1975, p. 141 and Tiella, 1980.

[14] Hellwig, 2015 presented measurements of the frets together with deviations of the fret positions from extended 1/4 comma meantone. The sizes of whole tones, major thirds and fifths were also calculated from the fret positions. My thanks go to to Christopher Stembridge for a copy of the 2015 data. I am also grateful to Klaus Martius for supplying me with data on the monochord from the museum's archives in July 2020, as well as for discussions on several occasions over the years concerning this monochord.

[15] Zarlino, 1558, p.130.

the diagram is not easy to understand. For comparison, and as the competing hypothesis, a 1/4 comma meantone division was also constructed on transparent foil.

The two divisions were calculated for the octave size 532 mm, ± 2 mm, so that it would be possible to find the position of minimum error for all frets, and even take into account the possibilty of inaccurate bridge positions.[16] The foil was laid over the museum drawing of the monochord and moved until the best fit resulted. However, the museum drawing does not correspond exactly to Hellwig's 2015 measurements so this was only taken as an initial assessment.[17]

Nevertheless the discrepancy in the ♭♭ frets is so large (mostly 20 to 30 cents flat) that the trained eye can detect the problem through the *pattern* of the layout (see Fig. 1). From this visual examination of the monochord it appears that mistakes could have occurred in the construction, which would have been errors of thinking, not of layout inaccuracy.

This visual test showed how badly the ♭♭ notes, as well as the F♭ and C♭, fit any temperament pattern, but it also showed that 2/7 comma meantone seems to provide a better fit than 1/4 comma meantone.

3. A numerical approach

Although the visual approach has some advantages for the investigator, it has the disadvantage that it is difficult to describe or quantify for other parties the deviations of the frets from any tuning.

The following numerical approach overcomes the difficulty of describing the amount of deviation from a meantone pattern. The ratio of the length of a string at the fret, with the C string, can be expressed in cents, which is the approach taken by Hellwig, so that we have a list of cents, as we would have for a temperament.[18] See Table 1, col. 6.

The cent values in Table 1 (cols. 4-5) show a fairly even 22- (on the minus side) to 18+ (on the + side) distribution of errors for 2/7 comma meantone, but (cols. 8-9) show a highly skewed 31- to 9+ distribution of errors for 1/4 comma meantone. This pattern of error distribution alone suggests that 2/7 comma meantone was more likely the basis of the division.

Averages (arithmetic means) of all the + and – errors when comparing the 2/7 with 1/4 meantone theoretical values do not yield a sufficiently clear indication, although on this analysis 2/7 comma meantone has slightly less average cent error:
2/7 comma = -6.3 / +4.8
1/4 comma = -6.7 / +5.1
The situation appears slightly different if one omits the 9 apparently defective ♭ and ♭♭ notes. Then 2/7 comma has -4.1 / +4.8 average cent error, whereas 1/4 comma has

[16] As already noted, the c fret indicates a 0.45mm discrepancy compared with the note C, an octave lower. Whether this error can be removed or minimised by considering a different position for either bridge is a more complicated issue, but the primary task is to establish the underlying tuning system.
[17] Handwritten measurements by Hellwig from the Germanisches National Museum archives show slight differences to those published by Hellwig in 2015; the latter are described as the mean of two sets of measurements from 1981.
[18] Hellwig, 2015.

-6.6 / +5.1, which seems to make an even stronger case for 2/7 comma, but we must be careful not to be led to a conclusion by such small cent differences.

At this stage some statistical tools might be invoked, such as regression analysis, in order to find the better fit of the data to a tuning system. However, what is required is some test which is specific to the meantone tunings. As this article will argue, an *understanding* of the tuning systems and enharmonic tetrachords is required in order to decypher the monochord; merely identifying the best fit to a tuning is not sufficient.

A characteristic of 2/7 comma is that the major thirds are slightly narrow (383.2 cents, 3.1 cents flat). Whereas most temperaments have wider thirds, 1/4 comma meantone has just thirds (ratio 5:4, 386.3 cents).

Although we could hardly expect to detect a 3 cent difference with any accuracy, given the average errors of 5 to 6 cents with regard to both systems, the distribution of major thirds measured from C towards e (down the list of Table 2), shows that of 32 thirds in the monochord only 9 intervals are larger than the just third. Of these 9, 5 are the defectively low ♭ (or ♭♭) notes, already noted above, which we would prefer to examine separately.

Although we know nothing of the method or order of construction, one could *prima facie* expect that the probable constructional errors would tend to balance themselves out. The average size of the remaining 23 major thirds (after excepting the defective flat notes) is 380 cents, obviously flatter than just 386.3 cent thirds. Had the division been made for 1/4 comma meantone one would expect a more even distribution of thirds above and below the just ratio (386.3 cents). The strong bias to thirds on the flat side of 386.3 cents again speaks for 2/7 comma meantone.

There is a further test possibility: 2/7 and 1/4 comma meantone temperaments contain a series of fifths which produce a large fifth ("wolf" fifth) at g♯ - e♭ of 746.1 cents (2/7 comma) and 737.6 cents (1/4 comma); a pure fifth is 702 cents. This 9 cents difference between the temperaments is large enough that one might be able to detect it as a fourth from e♭ - g♯ (12 steps on the monochord), despite inevitable errors of construction.

To test this possibility, the theoretical 12-step sizes, corresponding to the notes e♭ - g♯, were calculated in 2/7 and 1/4 comma meantone. See Table 3. This reveals that there is an unusual interval, a "quirk", which occurs whenever ♯ and ♭♭ notes, or ♭ and ♯♯, form the ends of the 12 steps. This quirk is 30 cents in 2/7 meantone, and is equivalent to the mismatch of the pitch of notes completing the circle of 31 fifths. We could call this "Zarlino's comma", by analogy with other "commas" resulting from the mismatching of pitches of other intervals. For example C♭♭ is 30 cents higher than A♯♯ in 2/7 comma meantone (Table 1A), leaving a "gap" in the circle of fifths. In 1/4 comma meantone, with wider fifths, the "gap" or "quirk" shrinks to only 6 cents.[19] In 31-note equal temperament [ETS 31] there would be no such "gap" or mismatch.

This 30 cent "quirk" yields 12-step sizes which leap from the normal 453.91 cents to the abnormal 483.9 cents with 2/7 comma meantone. In 1/4 comma meantone, the 6 cent "quirks" yield normal 462.4 cent 12-step sizes, but only 468.4 cents abnormal ones. Table 3, cols. 4-5 illustrate the position of normal steps in 2/7 comma, col. 5 the first abnormal

[19] This small, only 6 cent, discrepancy is the reason why 1/4 comma meantone has been recognised as nearly equivalent to 31-note equal temperament, discussed further in section 24.

step. For lack of space, col. 6 only lists the occurrence of the *start* of each abnormal 12-step sequence.

Although there are only two "quirk" sizes, which do not involve the problematical ♭ and ♭♭ notes, these two (C♯♯ - G♭ = 469.8 cents, and D♯♯ - A♭= 470.1 cents) average 16 cents larger than the "normal" 12-step size in 2/7 comma tuning. This is considerably more than the 6 cents of 1/4 comma meantone, so this test indicates 2/7 comma meantone.

There are 15 "normal" 12-step sizes, which average 454.7 cents. Only 2 of these are larger than the 1/4 comma "normal" 12-step size. Thus, the average size is clearly too small for 1/4 comma meantone (462.4 cents) and is close to the theoretical 2/7 comma meantone size of 453.9 cents. Both normal and abnormal 12-step sizes speak for 2/7 comma.

The numerical methods of testing for the method of division indicate that it is 2/7 comma meantone which is clearly the more convincing explanation as the tuning scheme intended for the Clavemusicum Omnitonum.

4. The construction method of the monochord

Having detected clearly defined intervals in the monochord, the general question arises, how could any accurate or consistent fret layout of a *temperament* be possible? After all, a hallmark of a temperament is that there is no whole number ratio with which one can describe the narrowing of the series of fifths. Accordingly, in Zarlino's method there is no ratio for the construction of *tempered* intervals.

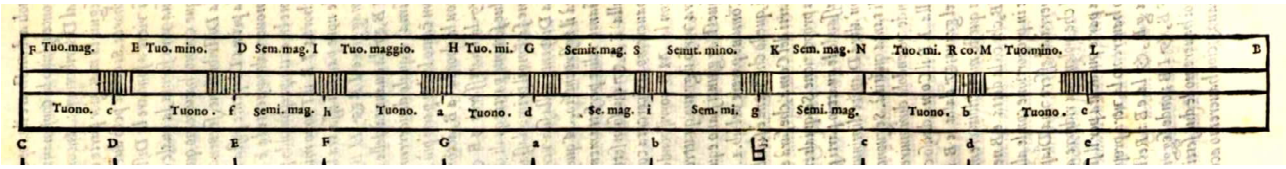
All the constructions are performed by adding or subtracting parts of the syntonic comma (ratio 81:80, 21.51 cents) to or from the appropriate intervals.[20] It is *not* a method involving calculation using ratios and numbers representing string lengths, such as we find on several other Zarlino monochords.[21] Zarlino's method anticipates logarithms, which permit multiplication and division through addition and subtraction, so the procedure is in essence the same as using cents to perform the calculation; it is merely that the technical means were lacking in Zarlino's time.[22] Lindley drew attention to his achievement in constructing the 2/7 comma tuning and characterised it as "mathematically correct " and "quantitatively exact".[23]

In theory, and on Zarlino's insistence in his dispute with his pupil Vincenzo Galilei, the syntonic comma should be divided into 7 geometrically-equal parts, not arithmetically into 7 parts.[24] However, the difference between the two methods would be only 0.005 mm in Trasuntino's monochord, indicating that Zarlino's displeasure with his former pupil had

[20] Airoldi, 1984 was probably the first to discuss Zarlino's tuning in detail, followed by Lindley, 1987 (in German) and Lindley, 1990, and 1997. Airoldi, 1989 is the published form of his earlier argument. I am obliged to Christopher Stembridge and Domen Marincic for obtaining a copy of this work, and for the assistance of Elena Ferrari-Barassi. Celhoffer also analyses Zarlino's tuning system.
[21] E.g. Zarlino. 1558, p. 117 on the enharmonic octave monochord.
[22] Logarithms were first applied in the field of music from 1639 by Cavalieri, according to Barbieri, 2008, pp. 282-285.
[23] Lindley, 1997, p.116.
[24] Zarlino, Sopplimenti musicali, pp. 189-191, criticised Galilei for using an arithmetical division of the syntonic comma, although he was never mentioned by name, only as "mio Discepolo". Zarlino was piqued that Galilei had described the 2/7 comma tuning without citing him as the inventor of it. Lindley, 1997, footnote 28, mentioned this briefly, but Celhoffer pp. 52-54 discussed the matter more fully.

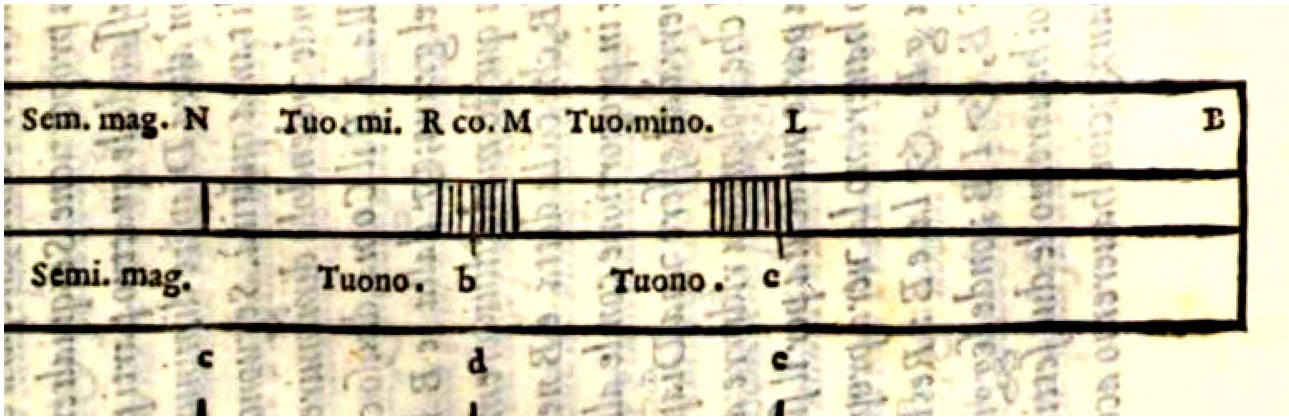
overcome his sense of proportion in practical matters. For example, in Trasuntino's monochord, the correct distance by which the F string should be shortened (compared with just intonation, in order to raise its pitch) is 1.46 mm, in order to achieve the +2/7 comma (6.15 cents) adjustment, so 0.005 mm represents a pitch 0.022 cents higher, resulting in a beat tone of 0.04 Hz, which is imperceptible and almost impossible to measure in an instrument, even today.

Zarlino did not provide scaled drawings of the exact division nor numbered string lengths. Instead we have a drawing, containing the syntonic comma at each tone or semitone, the comma being divided schematically into 7 parts.[25] (Table 4) This diagram is difficult to understand, partly because the sizes of the intervals drawn on the diagram are not proportional to their actual magnitude; one minor semitone is even shown as larger than a whole tone; a major tone (C-D) is drawn as smaller than a minor tone (D-E). Table 4A clarifies these matters of size. Here is the relevent part of the diagram (part of Table 4):



Source: Zarlino, 1558, p. 130 (part of Table 4)

The starting point on Table 4, shown below, is the syntonic comma which is defined by the two lengths RB and MB, yielding the distance between them of RM; this is shown in the top line of the diagram as "R co. M". Although "B" at the right of the monochord is not discussed, it follows that it must represent the note f.[26]



Source: Zarlino, 1558, p. 130 (magnified part of Table 4)

This comma is divided into 7 parts, assigning 3 of them to the minor tone c, on the high side (labelled "b" in the middle line of letters), and thereby creating the note d (no. 66 on the 1606 monochord). We should note that this method is thought of in terms of monochord string lengths; it is not an instruction for tuning the instrument by ear. However, having found a correctly-tempered c-d, we have not yet seen the route to find any other

[25] Zarlino, 1558, p.130.
[26] Zarlino, 1558, Cap. 42, p.126. The intervals major third (ratio 5:4) and Pythagorean minor third (ratio 32:27) form the distances RB and MB respectively, although Zarlino does not give these details. This is the most economical construction.

tempered intervals. Zarlino merely tells us in this chapter that all fifths should be be 2/7 comma narrow and the fourths widened by the same amount.[27]

In the following chapter, Cap. 43, Zarlino describes the determination of C - G, narrowed by 2/7 comma, which is to be divided according to Cap. 25 by a geometrical method, where a *mesolabio* is described.[28] Then a series of fifths and octaves follows: G > d, d > D, D > A, A > e, e > E, E > b quadro. At this point the direction reverses with C > F, and finally F > b molle, but there are no instructions here to find the accidental notes.[29]

An assiduous instrument maker might have executed Zarlino's geometrical tempering procedure for the comma with an entire series of fifths until all intervals had been tempered. It requires a high level of industry that might have been undertaken once, and probably then stored on a ruler from which all subsequent instruments would be built.

On examining the monochord it appears as if (initially following the order of Zarlino's description) a correct 2/7 comma division was found for d (fret 68), then correct sizes of C > G, followed by c > F.

Zarlino's temperament is now usually described as "2/7 comma meantone", thereby referring to the tempering of the *fifths*. Less often stated is that the minor semitone is not tempered, this interval having the ratio 25:24. Although the major third and minor third are both tempered, they are reduced by the *same* amount (-1/7 comma), as a result of which the minor semitone retains the 25:24 ratio, as in just intonation. A chart showing the tempering of the intervals in 2/7 comma meantone is given in Table 8.[30]

It is therefore possible on a monochord to derive the minor semitone by constructing a just intonation *major* third in one direction (ratio 5:4), then, from that fret, a just intonation *minor* third (ratio 6:5) back towards the starting point: the constructed interval is *exactly* 25:24.[31]

As if giving a hint to this procedure, Zarlino described how every string of the diatonic or chromatic genera should have strings for a major third (M III, ratio 5:4, *Ditono*) and minor third (m III, ratio 6:5, *Semiditono*), both above and below.[32] What this means is that with

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[27] The entire discussion is difficult to unravel because Zarlino uses three different designations in the drawing, the lower letters (C-e) being the note names of the 2/7 comma tuning. The top row gives the intervals in just intonation, but the lettering is different from the lower row; it corresponds to the monochord on p. 124 (right hand side). The middle row of letters is for the tempered notes and different again from the other two rows. Thus, the tempered notes at the comma divisions are given letters which do not correspond to their usual note names. With these opaque notations Zarlino demands considerable persistence from the reader. Airoldi, 1989 patiently dissects these instructions pp. 62-75, but the reader need not study each step in order to understand the principles involved. Table 4B shows all the manipulations of parts of the comma.

[28] Zarlino, 1558, pp. 95-96. As Lindley 1997, p. 182, observed, the use of a mesolabium is theoretically correct, but impractical.

[29] The size of the chromatic semitone in just intonation was dealt with later in Zarlino, 1558, Cap. 46, pp. 137-139. Only in Cap 47 do we find the necessary information about the division of the enharmonic tetrachord, which would have enabled an instrument maker to construct all the accidental notes.

[30] This approach, initiated by Celhoffer, p. 53, is helpful in understanding the tuning.

[31] This requires relatively little constructional effort beyond the first division of the string length into 5 parts. 4 of these are taken for the major third above (ratio 5:4). For the minor third below (ratio 5:6) it is not necessary to re-divide these four parts into 5 and then add one to reach a minor third below: one could simply take 0.8 parts of one step, perhaps with a proportional compass. This would make the procedure relatively simple compared with the direct 25:24 ratio, although even here the experienced practitioner could make use of a "pre-calibrated" division in the 12 1/2" : 12" of a normal ruler. See also note 107.

[32] Zarlino, 1558, p. 140, lines 19-20: "... ogni chorda diatonica, & ogni chromatica delli detti istrumenti, si verso il graue, come etiandio verso l'acuto, hauerà vna chorda corrispondente per vn Ditono, & per vn Semiditono..."

this two-step procedure, starting at C and constructing frets from, and then back towards C, the positions for C♯ and C♯♯ could be found exactly.

The practical issue is then how one constructs the semitones for the other whole tones. One can construct an E♯ from C♯ with M III, but this will be with a +3 cent inaccuracy because the just ratio (5:4) we are using is 1/7 comma too large. However, it is possible to navigate the whole monochord in this fashion, if one is prepared to accept occasional 3 cent errors, or 6 cents if taking a further step (e.g. E♯ - G♯♯). It is *not* possible to manoeuvre to the diatonic notes without error using these just ratios (5:4 and 6:5).

Of course, in order to find a diatonic note it would be possible to create correctly-tempered fourths and fifths, as Zarlino indicates. Indeed, it appears as if this has been done since F and G have only small errors, and in the correct directions (widening and narrowing respectively). From these F and G notes further semitones can be found, and ultimately the entire moncochord can be divided.[33]

This procedure with alternating major and minor thirds (M III and m III) reveals an interesting error which can occur, if one strays from the procedure I have outlined. If one attempted to continue the series from C♯♯ to G♭♭ then the resulting G♭♭ would be 30 cents too flat. This is because in 2/7 comma meantone there is a 30 cent gap between the series of fifths on the sharp side leading to B♯♯ and the series of fifths on the flat side leading to D♭♭ (B♯♯ = 20.39 cents, D♭♭ = 50.28 cents); this is the "quirk" discussed earlier. There is a simple rule to avoid this problem: if using the M III / m III method, one may not construct a ♭♭ note from a ♯♯ fret, or vice versa. Expressed in terms of the *ordine* (Table 5), one should not mix the *ordine B molle* side with the *ordine B quadro* side.

This M III / m III construction method, leading to the 25:24 semitone, shows us that despite layout inaccuracy, we could expect to find clear minor semitone sizes and thereby interval patterns of the 2/7 comma division which can be *distinguished* from 1/4 comma temperament. This seems to be why we were able to identify the monochord fretting as better fitting the 2/7 comma tuning using visual and numerical methods than 1/4 comma meantone.

5. Could the monochord division have been made with just intonation?

Although Zarlino's 2/7 comma tuning yields a better explanation of the monochord's construction than 1/4 comma meantone, it does not explain everything we find.

In Trasuntino's monochord the frets C – D (nos. 32 and 37) and D – E (nos. 37 and 42) yield slightly uneven sizes for the whole tone (202.3 and 183.3 cents respectively), which suggest their origin in the ratios 9:8 (203.9 cents) and 10:9 (182.4 cents), as if literally following the top line Zarlino's diagram (Table 4), where they are labelled "Tuo[no].mag[giore]" and Tuo[no].min[ore]" respectively.[34]

[33] It might appear as if 1/4 comma meantone with its perfect thirds would be easier to construct, but a similar problem occurs. From C one cannot reach all the notes with major thirds; it will be necessary to construct diatonic notes and proceed from them, theoretically with a geometrical method (e.g. mesolabium) for the meantone.

[34] Zarlino, 1558, p.130. Even Zarlino is unclear in his text, describing in the Dimostrationi, 1571, p. 221 that in his tuning there were two sizes of whole tone, an infelicity which he only corrected (without comment) in the 1589 L'Istitutioni edition, p. 242, as relayed by Barbieri, 2008, p. 15, note 19, following Airoldi, 1989, p. 57 and p. 106. Was this clumsy expression or was he misled by his own diagram which shows two sizes?

There are also other notes in the layout (consulting Table 1) which appear to show just intonation, again as if following the *top* line of Zarlino's diagram (Table 4), but without the necessary corrections from the comma.[35] The whole tone G (184.9 cents between frets 50 and 55) is close to a minor tone, and the tone A (196 cents between frets 55 and 60) closer to a major tone, just as is drawn by Zarlino.[36] These are tabulated in Fig. 2 below:

Fig. 2. Intervals of the monochord, just intonation and Zarlino's diagram, all values in cents

NOTE	C	D	E	F	G	A	B	c	d	e
Just intonation	0	203.9	386.3	498	702	884.4	1088.3	1200	1403.9	1586.3
monochord	0	202.3	385.7	509.5	699.4	884.3	1080.3	1197.1	1395.7	1582.4

Zarlino top line	203.9	182.4	111.7	203.9	182.4	203.9	111.7	182.4	21.5	182.4	
monochord	202.3	183.3	123.8	189.9	184.9	196	116.8	195.7		186.5	

However, what speaks against a complete just intonation scheme are the F and G fret positions, which are consistent with tempered fifths. Interestingly the c - d (195.7 cents) and d - e (186.5 cents) whole tones are somewhat closer to the correct 2/7 comma tuning value (191.62 cents).[37] We see apparently conflicting tendencies in the data: just intonation and a tempered tuning.

Thus, there are some grounds to infer that the maker of the 1606 monochord naïvely followed the top line of Zarlino's diagram where only just intonation interval sizes are given. Although this observation (if correct) would indicate a complete lack of understanding of Zarlino's tuning on the part of the monochord maker, it can be seen as circumstantial evidence that Zarlino's diagram was consulted.

Another problem should be addressed at this stage: the c length (no. 63 on the monochord) is not half the C (no. 32). In fact all of the notes above c are low in pitch, by between 2.7 and as much as 17.9 cents, with respect to the octave below. The entire history of monochord divisions demonstrates this practice of halving string lengths in order to find the octave. It is also surprising since it is one of the easiest constructions to ensure that the strings halve in length at their octave, regardless of whether the manufacture of the frets started at fret 32 (C) or fret 73 (c).

Inspection of the frets at c (no. 63, -2.9 cents) and e (no. 73, -3.5 cents) shows that an addition of 0.9 mm to the length of the C string (= 533.2 mm), i.e. moving the bridge 0.9 mm further away from the tuning pins (= +2.9 cents), would give a perfect correction for c and an excellent one for e (as used in Tables 3C-3H). Whether this represents a maker's error, or merely a difference in measuring method compared with the actual layout cannot be ascertained.[38] However, all the other frets from c - e would not be significantly altered

[35] Zarlino, 1558, p.126.

[36] One curiosity of the 2/7 tuning is that the just intonation major sixth (ratio 5:3, 884.36 cents) is so close to the 2/7 comma value (887.43 cents) that the just intonation major sixth is a completely practical construction for the monochord's note A, if we accept ± 3 cent errors as inevitable in the construction.

[37] If the c - d interval was in fact constructed with another just intonation *major* tone (9:8) on the monochord, then the actual fret represents an error of only 0.7 mm. It is therefore impossible to be sure what took place here. Zarlino uses a different procedure for the comma at d than an octave lower at D, although this is without particular significance.

[38] This discrepancy affects only the C string length since the other sounding string lengths from each fret to the bridge are unchanged. Bridge pins are generally around 1.0 to 1.3 mm in diameter so this factor might have something to do with the discrepancy. The sounding length of a string is nominally from the centres between the bridge and nut pins. Hellwig's museum data showed that he measured from C towards each fret, then calculated the sounding length, which lies between the fret and the bridge (at the right of Photo 3).

since the error there is just too large to be corrected by this method.[39] This means that there appears to have been some method of construction moving from fret to fret, thereby introducing this error.

This problem can be examined with another approach: instead of using cent values, as has been done until now, we can use absolute measurements in mm. The logic behind this approach is that an instrument maker would have been limited by his working methods in the accuracy he could attain. Whereas nowadays with conventional machine cutting techniques (i.e. without computer numerical control) an accuracy of ± 0.2 mm might be attainable it seems unlikely that a worker around 1600 could improve on this, if indeed achieve it.

Whereas the clavichord builder could bend tangents in order to achieve a satisfactory tuning, the maker of a monochord must work to exact dimensions if the tuning is to be produced accurately. Calculation shows that the difference between a just intonation major third, e.g. C - E, and a tempered third in 2/7 comma is 0.7 mm. This shows the accuracy of working which is required at the fret no. 42. Of course, an octave higher the difference is half of this, equalling 0.35 mm. This shows that an accuracy of manufacture which might be practically achievable of ± 0.5 mm would distort the tuning around E (fret 42) and render it audibly false at e (fret 73).

Tables 3C, 3D and 3E give the fret positions in mm required for 2/7 comma, just intonation and 1/4 comma meantone (respectively), together with the deviations in mm of the actual monochord from these hypothetical tunings. In this way one can show the error which the monochord exhibits with respect to each type of tuning. The central column for each set of measurements shows those falling within ± 0.5 mm, which one might consider *prima facie* to be the limits of constructional accuracy. 0.9 mm has been added to the string length at C in order to improve the octave equivalence at c, as discussed above. Thus, the C length (col. 5) on the tables 3C-3H is 533.2 mm (not 532.3 mm).

The results of this approach make it clear that the *whole tones* of the monochord are closely based on just intonation, as we had supposed from the analysis in Fig 2 above. In Table 3H, which compares the errors of the three tunings, this is clearly shown: the worst deviation from just intonation is only 0.8 mm (for B), and at three frets there is practically no error (i.e., no more that 0.1 mm). Given this clear correspondence to just intonation in the whole tones, we have also arrived at a record of the accuracy of work in practice which is -0.8 / +0.5 mm.

Table 3H also shows that the hypothetical correction for C to 533.2 mm improves the overall accuracy of the monochord. If one were to take the measured 532.3 mm then the errors from A to C are altered slightly but still remain within ± 0.5 mm. The errors become larger above A, reaching -1.1 mm at d.[40] Whichever C length is taken, the interpretation for just intonation whole tones remains correct.

There is however a slight change of interpretation as a result of this analysis with the corrected C length (533.2 mm): G is now seen to be *correct* in just intonation (only +0.1 mm), whereas previously it appeared to be tempered. This leaves us with only a diverging

[39] The errors are (without bridge correction): c♯ = -15, d♭ = -15.8, c♯♯ = -12.5, e♭ = -17.9, d♯♯ = -13.8 cents. Only d♯ would be correct after such a bridge movement.

[40] The actual errors are D = -0.4, E = -0.2, F = +2.6, G = +0.5, A = 0.0, B = +1.3, c = -0.4, d = -1.1 and e = -0.5 mm.

F note, which at +3.3mm is clearly beyond the limits of accuracy we can find in the monochord.

It is interesting to see that F has been made 5/7 comma sharp (with respect to just intonation), which is exactly the amount one can observe on Zarlino's diagram, as indicated by the letter "h", above the note "F" in Table 4. Of course, if this were actually the explanation of the F fret then it would show that the worker had misread the direction in which parts of the comma should be added. He should have made it 2/7 comma sharp. We would though still be at a loss to explain why only the F had been altered, and none of the other frets.

Although this makes a clear case for just intonation in the layout of most of the whole tones, the position of the frets for the ♯ and ♭ notes (of a 19-note compass) does *not* lead to the same conclusion. Here we find that the 2/7 comma tuning yields less error (7 notes with 0.5 mm error, or less; just intonation 1 note with 0.5 mm error, or less). (Table 3G)[41]

None of the three tunings considered shows a *systematic* pattern of error distribution for the ♯ and ♭ notes. Furthermore, even for 2/7 comma, 8 of the 16 notes have errors exceeding 1 mm, which is puzzling considering the accuracy achieved for the whole tones (Table 3G). In just intonation 11 of the 16 notes exceed 1mm error. The errors are also largest at the lower end of the monochord where greater accuracy could be achieved. C♯ is surprisingly far from its correct value (+3.5 mm), in any tuning. We are viewing a mistaken approach here, but it is unclear what the error of thinking was.

Since Zarlino's 2/7 comma tuning is only a modification of just intonation and retains the 25:24 ratio semitone, it is difficult to prise any distinction between the two tuning systems with statistical methods. However, the 12-step method, with which we could distinguish 2/7 comma from 1/4 comma meantone, gives us again some purchase on the problem.

With the 2/7 comma tuning there are 30 cent differences between some 12-step intervals. In just intonation this difference increases to 33 cents, but there are also two intervals 22 cents *smaller* than the majority 457 cent interval, the smaller size being due to the presence of the un-tempered major whole tone, starting at D♭ and D: this size is nominally 435 cents (see Table 3A). If we compare the monochord with the 2/7 comma tuning and the just intonation versions for the 12-step intervals, then we find that the monochord conforms to the 2/7 comma tuning. The monochord does not exhibit this unusual 435 cent size because the note F is tempered, as one would expect in a tempered tuning, and the F♯ and F♯♯ are correctly sized with respect to it, i.e. nominally 71 cents larger. The monochord's 12-step interval sizes are mostly closer to those of the 2/7 comma tuning, not the just intonation version.

Furthermore the comparison of the major third sizes shows that the majority are significantly smaller than pure (as we have already recorded), which also argues for the 2/7 comma tuning as the basis of the monochord construction, not just intonation. Thus, despite the elements of just intonation, the 2/7 comma tuning is the overall result we observe in the monochord.

[41] The tables 3B, 3D, 3E, 3F and 3I were part of the calculations for this section. They are not discussed in the text, but have been retained for the data they include.

6. Zarlino's diagram as the basis of the monochord

We have established that 2/7 comma meantone is the observed, resulting tuning of the monochord. There are also several of features which indicate that Zarlino's diagram was used as the basis of the monochord design:

(A) The monochord compass C-e
(B) Major and minor tones of the monochord follow the diagram's top line showing unaltered just intonation
(C) The c-e notes and the C-D are divided accorded to the diagram, i.e. differently
(D) F is altered by +5/7 comma, as if literally following the diagram

It is therefore concluded that Zarlino's diagram was the basis of the monochord design, which accords with the finding of the 2/7 comma tuning scheme. This confirms the intention to model the monochord on Zarlino's tuning scheme, even if it was poorly understood and inaccurately executed.

7. The discrepant flat frets

Since the level of understanding is not yet clearly established with which Zarlino's diagram was implemented, the question of the discrepancies in the ♭♭ notes of the monochord has a particular significance. If the intention behind the discrepant ♭♭ notes could be discovered, it might shed light on the entire project of the monochord.

The discrepancies in the ♭♭ notes of the monochord for 2/7 comma meantone are significant (flat by 12.7 to 31.3 cents) and perceptible to the trained eye, even without the aid of calculation. This can be seen in Fig. 1, p. 6 above, where the correct D♭♭ and E♭♭ positions have been indicated by red lines. The D♭♭ is so close to the C, only 19 cents higher, that it seems it could be a B♯♯. Following this note (no. 33 on the monochord) through a series of fifths in the direction of the ♯♯ notes shows that the apparent ♭♭ notes are *actually* all sharp notes, technically ♯♯♯ notes: after B♯♯ must follow F♯♯♯. Calculation of the ♯♯♯ cent sizes shows that 7 of these notes have an average error of +9 cents. If they are viewed as ♭♭ then the average error is –23.17 cents.[42] Thus, the average values show that these notes are nearer to ♯♯♯ notes than ♭♭ notes.

Thus, it would appear at first sight, that through a systematic error, the ♯♯ series was continued further into the fret positions which *should* be ♭♭. The difficulty with this interpretation is that the B♯♯ should have been preceded by E♯♯, which apparently does not exist in the Clavemusicum Omnitonum, so how could a series of fifths have been created?

In theoretical discussions of tunings it is now common to speak of a *series* of fifths, which is not without historical justification since Arnaut's early monochord description for Pythagorean tuning used such a constructional procedure.[43] At this stage we have to analyse the order of the procedure more closely. Consulting Table 5, in order not to lose the orientation, we can see that the route from the diatonic notes via B♯♯ with a series of

[42] The C♭ and F♭ are included in this average of – values.
[43] See Arnaut de Zwolle's instructions for a "compō fimbrie", used for the organ pipes and for the fretting in a clavichord, fol. 129r°; Le Cerf and Labande, pp. 14-16. As seen above, this was not Zarlino's procedure.

fifths leads us "off the bottom" of the table to connect with F♭♭ "at the top" of the table, a note which does not exist in the Clavemusicum Omnitonum. If we attempt to reach D♭♭ through the *ordine B molle*, i.e. from G♭, then we are at a loss to explain why C♭ should be in error, since constructing the fifth G♭ - C♭ does not present any obvious difficulty.

We thus have *two* puzzles to solve: an E♯♯ does not exist, and why should C♭ and F♭ be defective (-25.8 and -20.8 cents respectively)?

As we have seen above, the construction of the monochord could have proceeded quite simply through alternating M III and m III intervals (i.e. major third and minor third); a series of fifths was not necessary.[44] If the "D♭♭" was in fact constructed with an M III from G♯♯ we would have in reality a B♯♯, which would explain our flat D♭♭. Similiarly, if the A♭♭ was constructed from D♯♯, then we have a note which is again nominally 30 cents flat. From the "A♭♭" (in reality F♯♯♯) one would, with this method, produce a defectively-flat C♭. Thus, we might be able to explain the production of these frets as errors resulting from the inappropriate use of the M III / m III construction method and 2/7 comma meantone.

We can also note that with 1/4 comma meantone, when constructing frets with just major thirds, we would not be able to produce the errors at C♭ and F♭, which could be seen as further, indirect evidence, for the 2/7 comma tuning.

The solution to our two puzzles appears to be that the constructor of this division did not think in terms of a series of fifths, as we are now inclined to do when considering tunings. Instead he could have worked from note to note on the monochord starting at C, ascending towards B, constructing the intervals with M III and m III ratios.[45]

8. Why should errors occur?

Although this explanation supplies a mechanism for producing an error, it does not supply a motive. We need not suppose that all these operations were performed on the actual 1606 monochord. Indeed, it appears from practical considerations that a separate ruler would have been used. Firstly, because it offers the possibility of more space for constructional operations beyond the C and e notes, as well as for trial and error. Secondly, because Trasuntino had already constructed a similar instrument in 1601 with ♯♯ notes, but no ♭♭ notes. Moreover, this 1601 harpsichord had an E♯♯ and a B♯♯, but no ♭♭ notes.

This 1601 instrument no longer exists, but was described by Stembridge from Pesenti's introduction to his "Correnti...", where the blind musician related his encounters with the Dominico Pesarese harpsichord built for Zarlino in 1548, and one made by Vito Trasuntino in 1601.[46] Pesenti's descriptions of the tuning reveal the notes available in both instruments, so Stembridge was able to correct the widely-held view that Zarlino's

[44] There is a parallel to this procedure in 15th-century treatises on monochord construction which show that in practice fourths and fifths were used alternately, in order to remain within a narrower range, usually an octave or two. Adkins gives a large number of examples.
[45] I have worked through the series of steps involving upward and downward steps from C and then other diatonic notes. This requires in excess of 45 steps, octave transpositions not included, but the presentation is so complicated that little benefit is likely to accrue to the reader in giving it here.
[46] Stembridge, 1993, pp. 44-54. See also Pesenti, introduction, "A Professori di Musica". Pesenti's text is given by Rasch, pp. 77-81, and another translation of Pesenti with the assistance of Riccardo Pergolis.

harpsichord had a 19-note range, and also describe Trasuntino's 1601 keyboard.[47] Zarlino's harpsichord had a 24-note range; that of Trasuntino had 28 notes, which were described without any ambiguity and can be presented in Fig. 3.[48] This style of presentation has the advantage of making the harmonic structure with M III and m III intervals clear.[49] (The keyboard layouts are presented as photos 5 and 6 below).

Fig. 3. Zarlino 1548 and Trasuntino 1601

						B♯♯								
					G♯♯			D♯♯			A♯♯			E♯♯
				E♯			B♯			F♯♯			C♯♯	
			C♯			G♯			D♯			A♯		
		A			E			B			F♯			
	F			C			G			D				
D♭			A♭			E♭			B♭					
	F♭			C♭			G♭							

black = Zarlino 1548 harpsichord G♭ - A♯♯
underlined = additional notes in Trasuntino 1601 harpsichord F♭ - B♯♯

Such a (hypothetical) pre-existing 1601 division ruler would therefore have contained a B♯♯ and E♯♯. This B♯♯ appears to be our defective D♭♭ in the 1606 monochord; the D♭♭ is 31.3 cents flat. However if it had been constructed as a B♯♯ on the monochord then it is only 1.4 cents flat. The 1601 instrument also had an E♯♯. Fret 46 of the 1606 monochord is only 13 cents sharp of E♯♯, but 16.9 cents flat of G♭♭, which is again a defective ♭♭, although the issue is not so clear as with the D♭♭. Thus, it is possible, that the 1606 monochord was derived from the 1601 version and contained its B♯♯ and E♯♯ notes.

Continuing this hypothesis: since ♭♭ notes were not needed in the 1601 division, it could have appeared to a worker in 1606 as if he merely had to "fill in the missing gaps" from the 1601 division. This hypothesis implies that such a worker had lost track of the plan, or never understood it, if he used the M III / m III construction from the ♯♯ notes to establish the ♭♭ positions.

However, we must consider that there might not even have been a division made for the 1601 harpsichord. A worker might have produced a chart of the notes in the division, on paper, in order to understand the plan. Barbieri has found a record of one such division, albeit much simpler with only diatonic notes, which Vincenzo Colombi (the organ builder

[47] Zarlino, 1558 p. 141 illustrated a harpsichord with 19 notes in the octave thereby giving this impression. In the first edition Zarlino stated imprecisely that *all* the major and minor semitones were divided into two tones. Pesenti cited Zarlino's description of the tuning from the 1573 (2nd, revised) edition where Zarlino corrected himself that the tone was divided into four parts. All this was clearly brought to light by Stembridge, ibid. As established by Riccardo Pergolis (personal communication, 2002), the 24-note compass was first described by Apel, 1967, p. 476. It was then mentioned by Martin, p.6, note 4 in 1984. Stembridge was the first to publish a detailed examination. Some sources continue to describe Zarlino's harpsichord as having had 19 notes per octave, e.g. https://de.wikipedia.org/wiki/Gioseffo_Zarlino (accessed 13.12.2024). Zarlino's 1558 text is at pp. 140-141. In the second, 1573 edition it is at pp. 163-164.

[48] How the blind Pesenti could ascertain the tuning was cogently answered by Rasch, p. 48, who related that when Charles Burney saw the instrument in 1771 there were instructions for tuning the instrument on the back of the nameboard of Zarlino's instrument. It can be presumed that these instructions were read to Pesenti. There is perhaps the faint chance that the instructions remain with Burney's unedited notes.

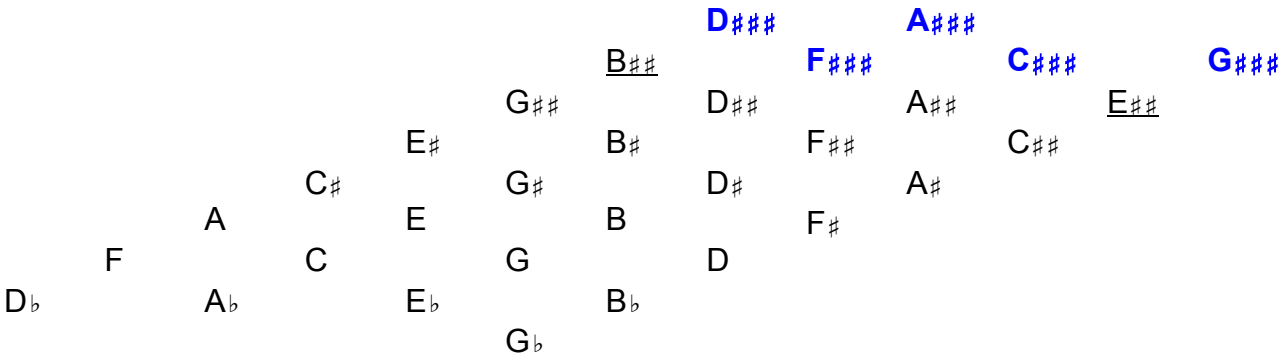
[49] This style of presentation is used by Barbieri, 2002, although he did not invent it.

praised by Zarlino) produced and sent to a mathematician.[50] However, the only practical reason for constructing a division is if a monochord is required for indicating the tuning. Of course, it is possible, and plausible, that for such a complicated instrument as the 1601 harpsichord, Trasuntino *also* provided a monochord. We can discern the didactic intent in the 1606 design: it is made with two pairs of strings and the fret positions are constructed with slots so that a removable bridge can be inserted into any slot. Thus, it could easily have been used to demonstrate any combination of two tones, or, with less easily plucked strings, the four notes of a tetrachord. Whether the monochord was accurate enough for tuning purposes is another matter.

The attractiveness of the explanation of the 1601 division as the origin of the 1606 monochord is apparently reduced by the fact that F♭ and C♭, both discrepant in the 1606 monochord, would also have been present in the 1601 division, at least according to Pesenti. Should we infer that the 1601 Trasuntino instrument was also defective in the F♭ and C♭ notes? This question is answered in section 18, but beforehand it is necessary to consider another matter.

Are the flattened ♭♭ notes really ♯♯♯ notes? This solution explains that the apparent F♭ and C♭, which are *in fact* slightly sharpened versions of A♯♯♯ and D♯♯♯ (+4 and +9 cents respectively), could be regarded as the triple sharp notes shown in blue, in Fig. 4 below. Is it possible that the Clavemusicum Omnitonum was actually conceived for the following range G♭ to A♯♯♯?

Fig. 4. Clavemusicum Omnitonum with extended range (**blue**) on the sharp side?



black = 1548 Zarlino
underlined = 1601 Trasuntino
blue [BOLD] = 31-note hypothetical range G♭ to A♯♯♯ of the Clavemusicum Omnitonum

Although such a scheme might seem initially strange, it is a continuation of Zarlino's own plan where in the 1548 harpsichord the range was extended on the sharp side (*ordine B quadro*) from B♯ with the 5 notes F♯♯, C♯♯, G♯♯, D♯♯, and A♯♯ (Table 5).[51] These provided the required major thirds for keys starting on "sharp" notes (F♯♯ for D♯ etc.), and in the same fashion the ♯♯♯ notes repeat the same procedure for keys starting on ♯♯ notes.

[50] Barbieri, 2002, p. 160. Zarlino, 1558 praised Vincenzo on p. 290, where he is described as "Vincenzo Colombi da Casal maggiore". In 1532 a document concerning the construction of an organ in Valvasone described him as "Vizenzo de Casalle de Monseva sta a Venetia", see Stella and Formentini, p. 8.

[51] Such ♯♯♯ notes are not as abstruse as one might think. A♯♯♯ and D♯♯♯ are required in descending tetrachords from B♯ and E♯ respectively in the Ptolemy-Vicentino system when tuned with 2/7 comma. Table 7/1A shows this, but other parts of the argument must be examined first.

One disadvantage of this arrangement would be that three of Pesenti's dances, those requiring C♭ and F♭ notes, would not be playable, but could Trasuntino in 1606 have taken account of Pesenti's work, published in 1645?

Although this solution appears to be possible on paper, could we test the likelihood through its realisation in Trasuntino's 1606 keyboard? The keylevers are numbered according to their pitch in the conventional way, the labelling of the keycovers and the monochord indicates that E♭♭ is lower pitched than D♯♯. Despite the apparently bewildering number of "split sharps", the hand position for the player remains the same, whether he plays C-E-G , C♯-E♯-G♯ or D♭-F♭-A♭. The ♯♯♯ interpretation would require us to place a C♯♯♯ to the *right* of the D key, which would surely require some presence of mind on the part of the player, even though the hand position is unchanged. The issue remains undecided, so it requires a better understanding of Zarlino's harpsichord and its development through his enharmonic system, (until a conclusion is finally reached in section 14).

9. Zarlino's enharmonic and his 24-note harpsichord

As Stembridge observed, Zarlino's instrument "was like a *cimbalo cromatico* with the addition of five extra keys: these were for F♯♯, C♯♯, G♯♯, D♯♯, and A♯♯.[52] According to Stembridge's earlier investigation, the 19-note "*cimbalo cromatico*" was intended for accompaniment, the extra notes overcoming the limitations of meantone tuning: for example, d♯ eliminates the g♯ - e♭ "wolf" fifth.[53]

Rasch could not find a motivation in *Le Istitutioni Harmoniche's* 'sketchy and indirect" description why five notes should have been added, so he asked why Zarlino needed a 24-note instrument (rather than a 19-note one) "to let him hear the enharmonic genus".

This question implied that we already "hear" the enharmonic with the 19-note harpsichord.[54] Van der Meer wrote of "partially" enharmonic instruments, so we need to understand what constituted the "enharmonic", as it was seen in the 16th century.[55] When was the function merely "chromatic", when did an instrument become "enharmonic", and which keys were needed?

According to Zarlino's definition, when discussing Boethius' rendering of the three Greek genera, the enharmonic genus is present where the semitone [*semituono maggiore*, diatonic, or larger chromatic] is split into two dieses.[56]

Thus, a 14-note instrument with "split sharps" for d♯ / e♭ and g♯ / a♭ already contains the enharmonic, according to this definition, since d♯ splits the d - e♭ large semitone. We now often use the word "enharmonic" to describe small intervals, such as the d♯ - e♭ interval

[52] Stembridge, 1993, p. 46. Despite the name "cromatico", such an instrument includes the enharmonic, as will become clear from the text below. One also finds "cembalo cromatico" in the literature, Zarlino writes of a "clavocembalo", not "clavicembalo", and Trasuntino writes "clavicimbano". There were many variants in the 16th century.
[53] Stembridge, 1992, p.9. The "wolf" is 737.7 cents in 1/4 comma meantone instead of the correct 696.6 cents.
[54] Rasch, p. 49. Rasch found that Zarlino's text was "sketchy and indirect" in this matter.
[55] See Van der Meer, 1987. The title is "Partiell und vollständig enharmonische..."
[56] Zarlino, 1558, p. 109: "...detto Enharmoniche: perche divide il Semituono, in due parti, cioè in due Diesis:"

(41.06 cents in 1/4 comma meantone), which is smaller than the chromatic semitone. It is obvious that the 16th to 17th-century use of the d♯ or a♭ in keyboard instruments has nothing to do with the Greek enharmonic, but is merely intended to provide correct intervals in the chromatic genus with a meantone temperament (or something similar). Zarlino's definition is imprecise, and we might disagree with it, but it was his statement.

As Stembridge explained, in the illustration of Zarlino's enharmonic harpsichord it was intended in the first edition of *Le Istitutioni Harmoniche* that the enharmonic keys should be red, in order to distinguish them from the chromatic ones.[57] See Table 4C. Although red was never used in the printing, it is clear that the keys Zarlino considered enharmonic were the "split sharps" not normally found in temperaments of his day, i.e. D♭, D♯, G♭, A♭, and A♯, with the addition of keys between E and F for E♯ and between B and C for B♯.[58] This means that despite the "sharp sign" we now give them, E♯ and B♯ were not "sharps", i.e. not part of the chromatic genus according to Zarlino's definition, but the *end* of the enharmonic genus (in the 19-note instrument depicted), when we trace the series of fifths out from B.[59] See Table 5, where the chromatic and enharmonic are ordered following Zarlino's definition.

Zarlino's practical definition is then a reflection of the contemporary use, wherein most music required no more than three sharp or two flat notes: the extent of this limit in the chromatic genus is E♭ and G♯, as seen in Table 5. When this limit was exceeded it was usual to include both D♯ and A♭, as is seen in the many Italian string keyboard instruments with 14 notes in the octave.[60]

Zarlino developed a notation for the enharmonic note, which is a simple diagonal cross (x). Pesenti noted that the printer did not have such a sign, and even normal, modern typefaces lack it. Instead I will adopt hereinafter red (following Zarlino's intention) for the indication of the enharmonic, but retain the "flat" or "sharp" sign since this tells us whether the note belongs to the *ordine B molle* or *ordine B quadro*, i.e. to the flat side or sharp side, respectively.[61] (Table 5)

In his description of "filling out" the enharmonic in his monochord, Zarlino cautions against adding notes having "little use", calling this vain and superfluous.[62] What constituted a "useful" note is indicated by his prescription that a string should create a fifth, fourth, major third, or minor third to another string, either above or below.[63] How this was implemented

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[57] Airoldi, 1989, p. 100. Stembridge, 1993, p. 45. Zarlino, 1558, p. 140: " quanto vaglia l'Arte aiutata dalla Natura, nel congiungere, & collocare mirabilmente, con bello, & regolato ordine le chorde Chromatiche tra le Diatoniche; & tra l'vne, & l'altre di queste, le Enharmoniche; Le quali si conosceranno nel Tastame delli detti Istrumenti in questo: che a differenza delle diatoniche, & delle chromatiche, si porranno di colore rosso; come nel sotto posto istrumento si può vedere.". The use of the colour red for these notes was first recorded by John Hothby c.1487; see Lindley, 1980, p. 15.

[58] Stembridge, 1993, p. 45 draws attention to this matter. In the 1573 (2nd) edition, Zarlino dropped the prescription for red and suggested simply "another colour". See Wraight, 2024, "Vicentino's..." p.29, note 97.

[59] Zarlino, 1558, p. 281, uses a single, diagonal cross to indicate an enharmonic note, but a double, diagonal cross for chromatic notes. Airoldi, 1989, p.100, considers E♯, A♯, and B♯ to be the enharmonic notes which can be found in the Greek systems.

[60] Wraight, 2016, gives a checklist of surviving instruments.

[61] This terminology has been adopted from Lanfranco. As one of the precursors of Zarlino he described the music theory which Zarlino would have learned.

[62] Zarlino, 1558, p. 141 "...poco utilità....senza dubbi sarebbe cosa vana, & superflua:". Stembridge 1993, p. 55, saw this as possibly an aside against Vicentino.

[63] Zarlino, 1558, p. 140: "Ma si debbe sempre auertire, come altre volte hò detto, che quelle chorde sono poste con qualche vtilità in vno istrumento, & in alcuno ordine, le quali sono in tal maniera collocate, che verso il graue, ouero verso l'acuto hanno vna chorda corrispondente consonante per vna Diapente, o per

can be seen in the following Fig. 5, with the enharmonic notes (in Zarlino's sense) now in red and the additional notes (beyond the 19 notes) red underlined.

Fig. 5. Zarlino 1548, 24-note harpsichord

								G♯♯				D♯♯				A♯♯							
						E♯				B♯				F♯♯				C♯♯					
				C♯				G♯				D♯				A♯							
		A				E				B				F♯									
	F			C				G				D											
D♭		A♭				E♭				B♭													
	[F♭]			[C♭]				G♭															

diatonic and chromatic = black
enharmonic 19-note instrument = **red** [BOLD]
enharmonic notes 24-note instrument = red [underlined]
[...] = enharmonic notes not in the 1548 harpsichord, but discussed in this article

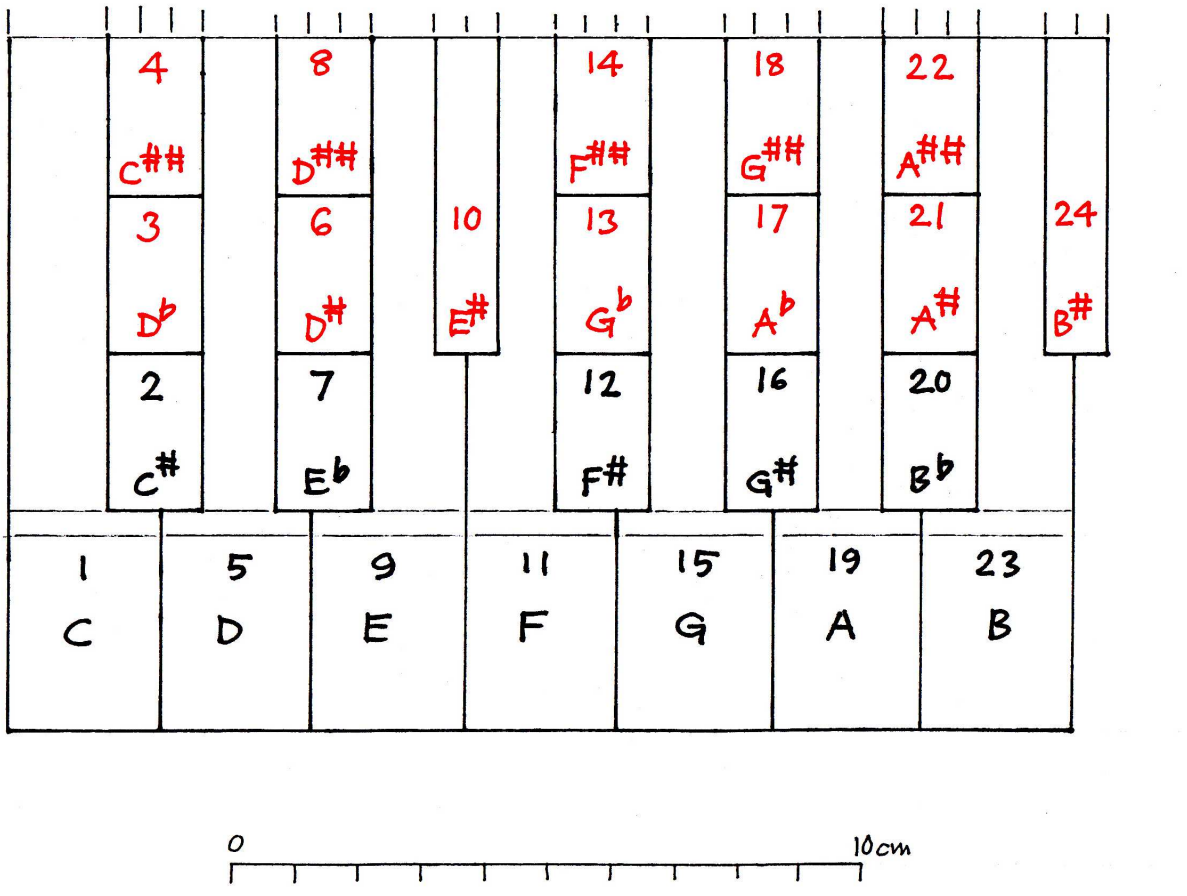

Fig. 5A. Zarlino's 24-note keyboard layout

vna Diatessaron, oueramente per vn Ditono, ouero per vn Semiditono; come sono quelle, che si ritrouano in questo istrumento."

As Stembridge calculated, following Zarlino's rules, the 1548 harpsichord should have contained the following 8 notes in addition to the 19-note *cimbalo cromatico*:[64]

F𝄪, C𝄪, G𝄪, D𝄪 (*ordine B quadro* side)
C♭, F♭, B𝄫, E𝄫 (*ordine B molle* side)

It is clear that following Zarlino's prescription, F𝄪, C𝄪, G𝄪, D𝄪 are required in order to provide the consonant intervals (M III / m III) for the four *already* existing enharmonic notes (D♯ - A♯ - E♯ - B♯) of the *cimbalo cromatico*.

However, the four notes required in the *ordine B molle* are missing, and the A𝄪 is harder to explain from Zarlino's indications. The visual symmetry of a 𝄪 note behind each raised key is an obvious practical consideration, but not discussed by Zarlino.

As Rasch noted, even the 19-note instrument permits *ascending* enharmonic tetrachords from all the diatonic notes, but Zarlino's 24-note instrument yields, in addition, *ascending* enharmonic tetrachords from the "raised keys" (C♯, D♯, F♯, G♯, A♯) (Table 6/1).[65] So this is the advantage the 24-note harpsichord confers over the 19-note instrument, but Rasch wondered what use this would have been to Zarlino, apart from permitting "full major scales on all ordinary sharps, and E♯ and B♯".[66]

The alternative explanation is that Zarlino was thinking in terms of ascending tetrachords, which would explain the presence of A𝄪. By adding merely five notes to the 19-note keyboard Zarlino doubled the number of possible tetrachords from 12 to 24.

We can expand the tetrachordal system beyond the seven diatonic notes to include all seven "raised notes", (♯ and ♭) for both ascending and descending tetrachords. Zarlino did not show us this, but it is an obvious inference. The number symbolism of seven would have been attractive for him. The result is Table 6/1 showing 42 possible tetrachords organised vertically, similarly to the *ordine B molle* and *ordine B quadro* sides.

Rasch has pointed to an important key in trying to understand Zarlino's intentions: we need to examine the instrument not simply harmonically, but also through the enharmonic tetrachordal system.

10. Zarlino's and Vicentino's enharmonic systems

There is an aspect of this matter which until now has not been considered. Zarlino's enharmonic system did not coincide with that of Vicentino, a notable pioneer, whose book *L'Antica Musica Ridotta alla Moderna Prattica* was published in 1555, three years earlier than Zarlino's *Le Istitutioni Harmoniche*. This difference of approach may not be well known and is emphasised here.

In introducing the enharmonic genus Zarlino described Ptolemy's tetrachord, but when he described his own enharmonic genus, with intervals on the monochord (65 pages later), he neglected to mention that it differed from that of Ptolemy, even though he usually cited

[64] Stembridge, 1993 p. 46.
[65] Rasch, p. 50. The "raised keys" which he listed (his expression for the "black" notes) did not include E♯ and B♯. The expression is apposite because it includes sharps, flats, and enharmonic notes.
[66] Rasch, ibid.

Ptolemy as an authority to be respected.[67] Vicentino used Ptolemy's version of the enharmonic genus.

Although Zarlino's enharmonic retains the 5:4 major third of Ptolemy's enharmonic, the two other intervals used by Zarlino are not known in any ancient Greek tetrachord.[68] Thus, Zarlino's enharmonic is not an historical "Greek" tetrachord at all, but devised from his priority to incorporate the intervals embedded in his *diatonico sintono* (diatonic syntonic monochord).[69] Zarlino's choice was also influenced by his attitude towards the enharmonic genus, as discussed in section 12 below.

The two tetrachord versions can be represented below (Fig. 6 and Fig. 7). The second line shows the dimensionless "string lengths" (368 etc.) as presented in the original, with cents included in the third line.

Fig. 6. Ptolemy enharmonico (= Vicentino), in Zarlino, 1558, p. 85 [PDF 103]

E	E♯	F	a
368	360	345	276
38.1	73.7	386.3 cents	
46:45	24:23	5:4	

Fig. 7. Zarlino enharmonico, in Zarlino, 1558, p.140 [PDF 158]

E	x	F	a
400	384	375	300
70.7	41.1	386.3 cents	
25:24	**128:125**	5:4	

Zarlino's tetrachord (Fig. 7) has the interesting property of containing a major diesis (ratio 25:24 or 70.7 cents), which is also part of a viable temperament. This cannot be said of the Ptolemy-Vicentino's tetrachord (Fig. 6) where the major diesis has the ratio 24:23 (73.7 cents).

If Zarlino had used Ptolemy's enharmonic system, then Ptolemy's 24:23 ratio would not have been compatible with his own 25:24 ratio for the minor semitone. This 25:24 minor semitone is found in Fogliano's just intonation monochord (Table 9), which Zarlino evidently incorporated in his 2/7 comma meantone temperament.[70] Although the tetrachord

[67] Ptolemy's enharmonic was introduced in Zarlino, 1558, as the last table on p. 85, then developed to an octave monochord on p. 117. How Zarlino gained access to Ptolemy's *Harmonics* is apparently not known. Palisca, 1985, p. 245, inferred that Zarlino knew some Greek, but needed the assistance of Antonio Gogava to translate Aristoxenus' *Harmonics* for him. Wraight, 2024, 'Vicentino's enharmonic lute...' p. 28, argued that Zarlino could have obtained all the information about Ptolemy from Gaffurio 1518.

[68] Chalmers' extensive list of enharmonic tetrachords, p. 171, with the characteristic interval of 386 cents does not give any ancient source for Zarlino's intervals 25:24 and 128:125. Chalmers attributes the 25:24 and 128:125 intervals in the enharmonic to Salinas, which is almost correct, but Zarlino had a prior claim, having published 19 years earlier.

[69] There was not a *single* Greek enharmonic system, but several. Those well known to Renaissance musicians before Gaffurio are to be found in Boethius, but Ptolemy's was not among them. Airoldi, 1989, p. 89, considered Zarlino's enharmonic tetrachord to be legitimate, in the light of classical theory. For further discussion of the transmission of Ptolemy's Harmonics see Wraight, 2024, 'Vicentino's enharmonic lute...'

[70] Fogliano's monochord was presented on fol. XXXIIIIv°. According to Vincenzo Galilei, who had studied music with Zarlino and might be expected to know, Zarlino had plagiarised the tuning from Fogliano's book without citing his source, although Zarlino claimed never to have seen it. Palisca, 1985, p. 246, showed how a section of Fogliano was translated from the Latin by Zarlino into Italian, which corroborates Galilei's version of the dispute. Zarlino's monochord, 1558, p. 124, has the same intervals as Fogliano's, albeit with numerical values at 1/5 of Fogliano's size, which makes this correspondence less obvious. The larger values Zarlino

is conceived in just intonation, Zarlino's other two ratios are also found in the 1/4 comma meantone *temperament* (128:125 and 5:4).

This departure from Ptolemy's enharmonic is the crucial point in Zarlino's entire tetrachord and tuning systems. Zarlino preferred a tuning system which retained the just intonation size of the chromatic semitone; this became his 2/7 comma meantone.[71] (See Table 8A for the structure of the 2/7 comma tuning).

However, the difference between the Zarlino and Ptolemy tetrachords is not merely that different chromatic semitone sizes are involved, but that Zarlino places the smaller (minor) diesis (mD) at the *second* position, a practice of which Ptolemy disapproved.[72] Vicentino followed Ptolemy's tetrachord, with the minor diesis placed first.[73] As Wild has observed, in this way Vicentino made it possible for the listener to hear at the *second* note in a piece of music that the enharmonic genus was involved.[74] Thus, Vicentino's and Zarlino's enharmonic systems are not identical, a detail not been widely recognised.[75]

Zarlino: MD – mD – M III (Tables 6/*) (Major Diesis – minor Diesis – Major third)
Vicentino: mD – MD – M III (Tables 7/*) (minor Diesis - Major Diesis – Major third)

We should examine the enharmonic genus firstly through the ascending tetrachords which Zarlino uses; these were presented 141 pages later in *Le Istitutioni harmoniche*.[76] Here are Zarlino's examples, reading left to right:[77]

Fig. 8.

→ MD	mD	M III
B	B♯	C E
E	E♯	F A
A	A♯	B♭ D

uses are probably the result of a wish to avoid fractions of numbers, rather than to conceal the source. In Zarlino's Sopplimenti musicali a C value of 14400 is used to avoid these fractions (see Table 4B).

[71] Zarlino was aware of 1/4 comma meantone: he described it in his Dimostrationi, 1571, p. 221 ["212" is incorrectly printed as the page number], but in more detail in the second (1589) edition, pp. 259-263.

[72] Ptolemy, Book 1, [33.22]-[33.24]. This passage was not available to Renaissance scholars through the well-known Boethius, but only from the original Harmonics, which Zarlino evidently knew, probably through Gaffurio; see note 67.

[73] This desciption of the enharmonic is that introduced by Vicentino, Book 1, Cap. 8, fol. 14v°-15r°. See also Wraight, 2024, 'Vicentino's enharmonic lute...', where Vicentino's chromatic tetrachord is found less elegant than Zarlino's. For further discussion of Vicentino's use of the enharmonic see Wild, Fig. 5.

[74] Wild, note 27.

[75] Chalmers, ibid., did not detect it. Rasch, p. 59, notes the difference, but he did not draw any inferences from it. For a more detailed discussion of Vicentino's system see Wraight, 2024, 'Vicentino's enharmonic lute...'

[76] Zarlino, 1558, p. 281. Zarlino's enharmonic table from the first edition (1558) was reproduced by Pesenti, although Pesenti omitted the grouping into tetrachords with their Greek names. Pesenti quotes Zarlino's description of the harpsichord from the 1573 (2nd) edition, which contained significant changes in the text, also enabling us to understand the 24-note instrument's range. Zarlino, 1558 also describes a descending tetrachord on p. 85, so the idea was not unknown to him: "Ma procedende d'all acuto al grave per l'istessi intervalli..."

[77] Zarlino presented his tetrachord for the monochord in vertical fashion, 1558, p. 140, and on p. 85 similar to Ptolemy. The horizontal form is preferred here, partly for reasons of space, but mostly because it accords better with the layout of a keyboard.

A consequence of Zarlino's order of the MD and mD is that ascending (F♯ - B♯) and descending (G♯ - B♯) tetrachords require in total 10 ♯♯ notes when we consider the 42-tetrachordal system (Table 6/1). Vicentino's system with ascending tetrachords (F - A) and descending (F♯ - A♯) requires 10 ♭♭ notes. Thus, Zarlino's apparent preference for increasing the chromatic range on the "sharp side" is inextricably linked with his "un-Greek" ordering of the MD and mD in his enharmonic tetrachord.

When we take the "flat notes" into account then we see that Zarlino's system also needs ♭♭ notes in 12 positions (Table 6/1, F♭ - B♭ descending, F♭ - A♭ ascending), but here Vicentino's system needs still more: ♭♭ notes in 15 positions (Table 7/3: F♭ - A♭, F - A ascending; F♯ - A♯ descending). Thus, the two enharmonic systems lead to completely different requirements regarding the notes which have to be supplied by any keyboard.

Zarlino's 1548 harpsichord with its range of G♭ - A♯♯ needed only 24 notes per octave with his enharmonic scheme, but would have required 31 notes for the same 24 tetrachords in Vicentino's enharmonic. The extra seven notes are E♯♯, B♯♯, F♯♯♯, C♯♯♯, G♯♯♯, D♯♯♯, and A♯♯♯ (Table 7/1A). These would have required a fourth bank of "raised notes", as in the Clavemusicum Omnitonum (Fig. 11A), just in order to play the same 24 tetrachords. Zarlino's enharmonic system clearly yields a more practical keyboard.

However, a simple counting excercise does not penetrate to the heart of the matter. Just as Zarlino created consonant notes in the *ordine B quadro* for F♯, C♯, and G♯, his own principles of consonant intervals would have required that he should have done the same for the E♭ and B♭ in the *ordine B molle*, with F♭ and C♭. Zarlino prevented this through his stipulative definition that the major semitone (MS) should be divided into *two* parts. He would have needed to divide the MS into three parts (i.e. provide two keys between E - F and B - C) in order to include F♭ and C♭. His system apparently gave priority to E♯ and B♯ in order to provide M III intervals for C♯ and G♯ (see Fig. 3). Descending tetrachords from C and F are also not provided for (Table 6/1), so a complete enharmonic is not possible, not even from all diatonic notes.

Even with this better understanding of Zarlino's tetrachord system we cannot yet explain all the choices he made with his 24-note instrument. It requires an understanding of the shortcomings of the 2/7 comma tuning to elucidate this.

11. Consequences of the 2/7 comma tuning for Zarlino's 1548 harpsichord

Through Zarlino's presentation in *Le Istitutioni Harmoniche* it is clear that his 2/7 comma meantone tuning and his enharmonic system are in some way linked, so that we should see his 2/7 comma temperament as intended for the 1548 harpsichord. This option was not an aesthetic choice, but as we can now discover was guided by the structure of Zarlino's tetrachords and the 2/7 comma tuning as a version of just intonation, with its unaltered minor semitone (25:24 ratio).[78]

[78] Lindley, 1987, p. 158-159, linked Zarlino's 2/7 comma tuning with the diatonic, chromatic and enharmonic monochord, but the 1548 harpsichord with 19 notes in the octave [sic.]. Airoldi, 1989, and Sukljan described the tuning of the enharmonic whereas Lindley did not.

Zarlino did not commit himself to a fully-enharmonic instrument, nor even one with the harmonic complements to the usual two flats (e♭ and b♭) of a normal (12-note, chromatic) instrument. This has been the most difficult part to explain since on the one hand Zarlino criticised those who provided extra notes having no use (in his view), but on the other hand he failed to provide the consonant notes (F♭ and C♭) for the *ordine B molle* which his own theory required.

However, as Table 6/1 shows us, with the 2/7 comma temperament only 24 correctly-tuned tetrachords of the 42 are possible. There is therefore a technical explanation for the exclusion of F♭ and C♭ since descending tetrachords in Zarlino's 1548 harpsichord would lead to a mistuning with D♯♯ and A♯♯ respectively on his *keyboard* (Fig. 9); F♭♭ and C♭♭ would be required (lower two lines of Fig. 9).

Ascending tetrachords from F♭ and C♭ would not be possible in the 1548 instrument since G♭♭, B♭♭ and D♭♭ were not provided, (Fig. 9).

Fig. 9.

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	-30c D♯♯	+30 F♭		F♭	F		
G♭	B♭	-30c A♯♯	+30 C♭		C♭	C		
actual notes ↑					missing notes ↑		↑	
required notes ↓					required notes ↓			↓
C♭	E♭	F♭♭	F♭		F♭	F	G♭♭	B♭♭
G♭	B♭	C♭♭	C♭		C♭	C	D♭♭	F♭

KEY
+ oversize intervals (in cents, red)
- undersize intervals (in cents, blue)
enharmonic notes (in red, Zarlino's definition)

We see that the major diesis (MD, = minor semitone, ratio 25:24) intervals should be 70.67 cents, but become 30 cents larger (to 100.56 cents) instead, with a corresponding reduction in the minor Diesis (mD). This is the (approximately) 30 cent "quirk" ("Zarlino's comma") which we met earlier, since (for example) A♯♯ is 29.89 cents lower than C♭♭ in 2/7 comma meantone, (Table 1A).

As observed above, for the correct realisation in the descending tetrachords from F♭ and C♭, the notes F♭♭ and C♭♭ respectively would have been required. This would have posed significant practical problems in accommodating four notes in the keyboard *between* E and F, i.e. (in ascending order) E - F♭♭ - F♭ - E♯ - E♯♯ - F, and the corresponding notes between B and C: B - c♭♭ - c♭ - B♯ - B♯♯ - c. Note that E♯♯ is higher pitched than F, and B♯♯ is higher pitched than c.

Assuming that Zarlino understood the lack of pitch equivalence of D♯♯ ≠ F♭♭ and A♯♯ ≠ C♭♭, [≠ signifies "not equal to"], we can infer that his strategy in the choice of additional enharmonic notes was to avoid exposing the limitations of his tuning system.

Table 1A shows this A𝄪 ≠ C𝄫 difference using cents, but Zarlino could have calculated these differences along the lines of a table he published in the *Sopplimenti Musicali* for just intonation (Table 4B).[79] In order to avoid fractions of numbers he would have needed to multiply the C value of 14400 by 50, yielding 720000. This would have given him a C𝄫 = 390625. His A𝄪 would (with an arithmetical approximation for the 1/7 comma increase of A) have been 415452 (with rounding). For Zarlino there would have been a clear result that A𝄪 was not equal to C𝄫, but he would surely have recognised this anyway through his understanding of just intonation, without recourse to calculation.[80]

Thus, Zarlino's *complete* 42-tetrachord enharmonic system would require a 35-note range B𝄪 - F𝄫 for correct tuning with his 2/7 comma tuning (see Table 5, Enharmonico column, and Table 6/1A). The lack of notes on the *ordine B molle* side can be explained by the 2/7 comma tuning: the 24-note range A𝄪 - G♭ is all that is possible on the "flat" side without the mistuning of tetrachords. This is a subtlety of Zarlino's theory which until now appears not to have been described.

12. Zarlino's attitude towards the enharmonic

If Zarlino had never published details of his enharmonic harpsichord then we would have focussed on his views expressed in Part 3 of *Le Istitutioni Harmoniche* where he described the enharmonic [minor] diesis (ratio 128:125) as completely alien to the intervals used in counterpoint.[81] This partly explains Zarlino's antipathy towards a fully-enharmonic harpsichord: the enharmonic was for Zarlino not a genus for independent composition.[82]

As if more emphasis were needed, he criticised the "*Chromatisti*" in his final chapter, affirming his standpoint that good counterpoint could only be made from superparticular ratios.[83] This criticism excluded his own enharmonic from "good counterpoint", where the interval of the diesis had the ratio 128:125, which is not superparticular.[84] However, it is ironic that one of those he criticised anonymously, Vicentino, had already adopted the superparticular ratio 46:45 for his enharmonic diesis.[85] Of course, Zarlino was thinking of superparticular ratios with numbers no larger than the 25:24 he used in his monochord, but essentially those of his *senario*: 2:3, 3:4, 4:5, and 5:6.[86]

After this review of Zarlino's opinions, one wonders why he had an enharmonic harpsichord built at all. The answer is that it mostly served the purpose of a *cimbalo cromatico*, which as Stembridge argued, was intended to enable accompaniment.[87] At the

[79] Zarlino, Sopplimenti musicali, p. 155.
[80] Of course Zarlino would never have disclosed the arithmetical approximation of the 1/7 comma because he lambasted Galilei for using this theoretically incorrect method (see note 24), but the error is minimal.
[81] Zarlino, 1558, p. 288, "...il diesis Enharmonico, il quale è in tutto fuori di ogni proportione con gli intervalli positio nel contrapunto."
[82] See especially Zarlino, 1558, Cap. 73. Here Zarlino also drifts into a slightly different argument, that one cannot compose in the chromatic and enharmonic genera alone, in which respect he effectively supports Vicentino's position in the Lusitano versus Vicentino dispute.
[83] Zarlino, 1558, Part 3, Chapter 80. A superparticular ratio, is the ratio of two *consecutive* integer numbers, e.g. 2:3. In some older writings, such as Crocker, the term "epimore" is used.
[84] This is Zarlino's enharmonic as described with a *tetrachord* on p. 140. This enharmonic interval 128:125 is the same size in 1/4 comma meantone (41.06 cents), but in Zarlino's 2/7 comma *temperament* it becomes 50.28 cents.
[85] See Wraight, 2024, 'Vicentino's enharmonic lute...'
[86] Zarlino, 1558, pp. 27-28. Palisca, 1985, pp. 247-249, gives a summary of Zarlino's *senario* theory.
[87] Stembridge, 1992, p. 9. Instruments with fewer than 19 notes per octave should probably be seen as enabling transposition, not as being "partially enharmonic", an expression coined by Van der Meer. A 19-

same time, Zarlino could apparently not resist the temptation to link and legitimise his own tuning system with an ancient Greek heritage. Nevertheless, we should not belittle his achievement in keeping practical music close to the Platonic number ratios which formed the basis of music and of the structure of the universe, as it was then seen.[88]

13. Trasuntino's 1601 harpsichord

Equipped with this understanding of Zarlino's enharmonic system and the restrictions imposed by the 2/7 comma tuning, we can now return to Trasuntino's instruments. Trasuntino's 1601 harpsichord was not the first instrument he made with split sharps. A harpsichord of 1591 has survived, which was originally provided with a 19-note compass, and was therefore a "cimbalo cromatico" in Stembridge's sense.[89]

The 1601 instrument extended the range of Zarlino's harpsichord, and by including 4 new notes, C♭, F♭, E♯♯ and B♯♯ (which Pesenti clearly described), it made possible a further 6 enharmonic tetrachords (Table 6/2). Perhaps more significantly, these additions permitted ascending *and* descending enharmonic tetrachords from all the diatonic notes, as well as the "raised notes" in the *ordine B quadro* (F♯, C♯, G♯, D♯, A♯, E♯, B♯). In addition, ascending tetrachords on E♭ and B♭, were now possible, (Table 6/2).

Thus, we could see this instrument as a logically-consistent completion of Zarlino's system in the realisation of enharmonic *tetrachords* in the *ordine B quadro*, but it was also an extension of the system in the *ordine B molle* to permit tetrachords from the normal two flats, E♭ and B♭, which Zarlino's theory required, but his own 1548 harpsichord lacked. Thus, the 1601 harpsichord *appears* to be the instrument Zarlino should have had built for him.

Fig. 10. Trasuntino's 1601 harpsichord, extending Zarlino's 1548, 24-note instrument

										B♯♯													
							G♯♯					D♯♯				A♯♯						E♯♯	
					E♯					B♯				F♯♯				C♯♯					
			C♯				G♯					D♯				A♯							
		A			E					B				F♯									
	F				C					G				D									
D♭		A♭			E♭					B♭													
	F♭				C♭					G♭													

diatonic and chromatic = black (Zarlino 1548 and Trasuntino 1601)
enharmonic = **red** [BOLD] (Zarlino 1548 and Trasuntino 1601)
Trasuntino 1601 additional enharmonic notes = blue [underlined]

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note harpsichord (cimbalo cromatico) was made by Vito Trasuntino in 1591 (Wraight, 1997, Part 2, p. 302, W618 in my catalogue). I am grateful to Christopher Nobbs for extensive documentation of this instrument, which I had not been able to examine.

[88] In this regard it should be recognised that Zarlino placed the astrological symbols of the *seven* planets on the same chart as his diagram for the 2/7 comma tuning, but deleted them from the 1573-1589 editions.

[89] Now in the National Music Centre, Calgary, Alberta, Canada. The original compass was reconstructed from the balance rail on the keyframe by Christopher Nobbs (personal communication and drawing); the keyboard has not survived. A less ambitious harpsichord, W366, Schloß Köpenick, Berlin, had two split sharps in each octave and was probably made by Vito Trasuntino c.1559. None of the virginals with split sharps, which now bear Trasuntino's name, was made by him. See Wraight, 2016.

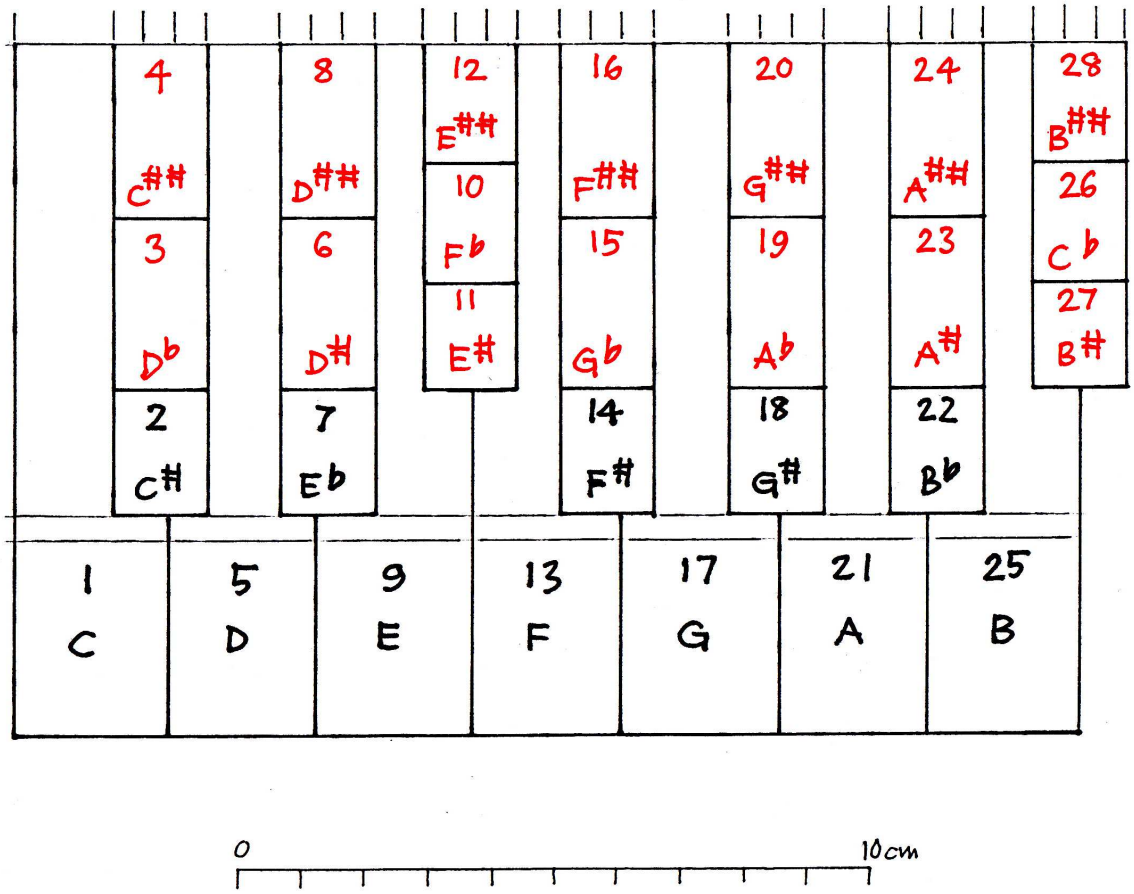


Fig. 10A. Trasuntino's 1601 harpsichord keyboard layout.
(The E♯♯ and B♯♯ have been numbered according to their keyboard position, not according to their pitch)

The tuning difficulty in the tetrachords from F♭ and C♭, which Zarlino skillfully avoided was necessarily present in Trasuntino's 1601 harpsichord, assuming that it was tuned in 2/7 comma.

The use that Pesenti made of the instrument with his collection of dances does not appear to help us much in understanding how Trasuntino conceived this instrument. Pesenti took advantage of the C♭ in order to include 3 further flat keys in his transpositions, but he made no use of the E♯♯ and B♯♯ keys; this might have required him to include an additional set of keys starting on ♯♯ notes.[90]

Whether Trasuntino included the E♯♯ and B♯♯ notes in order to make a set of scales possible from all ♯♯ notes, or in order to complete the set of ascending tetrachords F♯ - B♯ is hard to discern, but it is an interesting fact that E♯♯ and B♯♯ are required in Zarlino's enharmonic in order to complete all the tetrachords in the *ordine B quadro* from F to B♯ (Table 6/2). This is a coherent system, as a comparison of Table 6/1 (1548 harpsichord) and Table 6/2 (1601 harpsichord) shows. It also shows us the significance of the analysis

[90] Dances 1, 8 and 11 in A♭ minor, E♭ minor, and G♭ major respectively. F♭ is also required in A♭ minor. Pesenti called the transpositions of the first 11 dances "*cromatici*" although they include what are enharmonic notes according to Zarlino's definition.

through the tetrachords, as a means of understanding the range of enharmonic instruments, rather than simply through the harmonically-complementary notes.
By contrast the role of the E𝄪 and B𝄪 notes is different in the Ptolemy-Vicentino system (F♯ and C♯ descending in Table 7/4) and the overall pattern of possible tetrachords is completely different.[91] It is only possible to form tetrachords from 9 of the 14 diatonic notes.

The 1601 harpsichord did not provide tetrachords for all of the enharmonic notes in the *ordine B molle*, nor indeed even descending tetrachords for E♭ and B♭, for which 𝄫 notes would have been required. Therefore, Trasuntino's 1601 harpsichord went to the limit of what was possible with a 4-fold division of the tone. It would have required a 5-fold division of the tone to provide the 𝄫 notes and thereby extend Zarlino's enharmonic to all the notes of the *ordine B molle*.

14. The range of Trasuntino's 1606 Clavemusicum Omnitonum

Compared with Zarlino's 1548 harpsichord, Trasuntino increased the enharmonic capabilities of the 1601 instrument and brought the number of correctly-formed tetrachords from 24 (1548 harpsichord, Table 6/1) to 30 (1601 harpsichord, Table 6/2). How the range of the 1606 Clavemusicum Omnitonum can be described will now be examined.

We have seen that there are three interrelated factors:

1. the range of the keyboard (or enharmonic system)
2. the tuning
3. the order of dieses in the enharmonic system

Thus, we cannot evaluate, or perhaps even name, the notes of a *keyboard* without knowing for which enharmonic system and tuning it was conceived.

Regarding *Zarlino's* enharmonic system: although the addition of five fifths (F𝄪 - A𝄪) on the sharp side in order to create an hypothetical F♭ - A𝄪 range (Fig. 4 in Section 8) creates a self-consistent explanation, this alone does not justify its correctness. This hypothetical range F♭ - A𝄪 would have no useful purpose in *Zarlino's* enharmonic since the 𝄪 notes are not needed to *complete* any tetrachords starting on the diatonic, chromatic or enharmonic notes, as Table 6/1 reveals.

Regarding the *Ptolemy-Vicentino* enharmonic, the group of tetrachords (F♯ - B♯) in Table 7/1A shows us that 𝄪 notes would indeed be required for the correct rendering of descending tetrachords (F♯ - B♯) with 2/7 comma tuning. Table 7/1A also reveals that 𝄫 notes would also be needed on the flat side (B♭ - F♭); this would require a prodigious 45-note range of G𝄫 - A𝄪 with the 2/7 comma tuning.

In principle a 31-note instrument according to this 𝄪 hypothesis is possible with the *Ptolemy-Vicentino* enharmonic, but still has defects: whereas all the descending tetrachords starting on F♯ - B♯ would be correct (Table 7/1A), all the seven ascending tetrachords on the flat side ascending, B♭ - F♭, would still be subject to the 30 cent quirks

[91] Whereas the Ptolemy-Vicentino enharmonic requires E𝄪 and B𝄪 for descending tetrachords from F♯ and C♯ respectively, it also requires 𝄪: see bottom of Table 7/2. In Vicentino's practice all of these 𝄪 are replaced by 𝄫 notes.

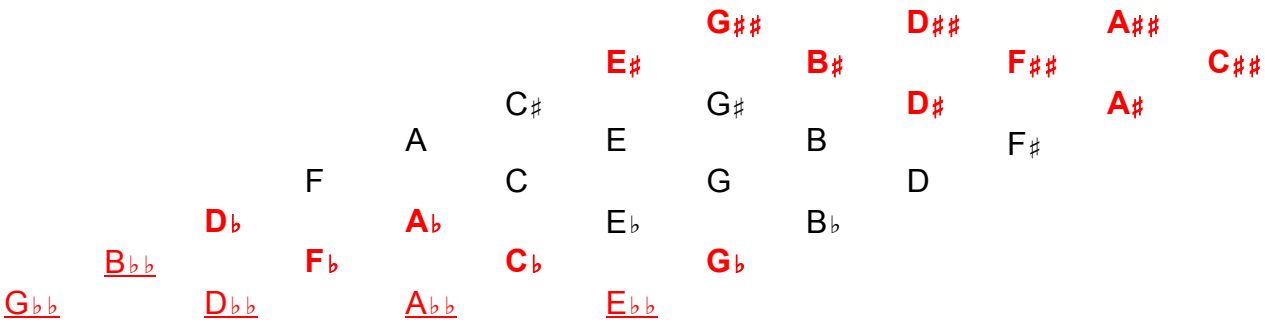
for 2/7 comma tuning (Table 7/1A) because the ♭♭ notes, and F♭ and c♭ would be lacking. There would be 35 correct tetrachords, still three fewer than with Zarlino's enharmonic.

The ♯♯♯ hypothesis (Fig. 4 and section 8) has no purpose in Zarlino's enharmonic scheme and has the weakness of still producing seven defective tetrachords in the Ptolemy-Vicentino enharmonic tetrachords, so it can now be clearly rejected.

The range of the 1606 instrument is usually considered to be that of Fig. 11 below, such as given by Van der Meer, having a balanced complement of ♭♭ and ♯♯ notes with the 31-note harmonic range G♭♭ to A♯♯.[92] Stembridge noted the omission of the 1601 harpsichord's E♯♯ and B♯♯ in the Clavemusicum Omnitonum, but observed that these were replaced by the G♭♭ and D♭♭ respectively, which he considered could be equivalent from the point of view of the tuning.[93] A later, dissenting voice was that of Barbieri who expressed caution in assigning this range of G♭♭ to A♯♯.[94]

With the range G♭♭ to A♯♯ 38 tetrachords are possible in Zarlino's enharmonic and 2/7 comma tuning (Table 6/3), but only 28 in the Ptolemy-Vicentino enharmonic, with the same tuning (Table 7/1). This shows us the significance of the monochord in revealing the intentions behind the planning of the Clavemusicum Omnitonum.

Fig. 11. Trasuntino's 1606 Clavemusicum Omnitonum (with 1601 predecessor)



diatonic and chromatic = black
enharmonic 1601 harpsichord = **red** [BOLD] (G♭♭ and D♭♭ instead of E♯♯ and B♯♯)
enharmonic additions 1606 clavemusicum omnitonum = red [underlined]

[92] Van der Meer, 1993, p 146-147. This text of the catalogue is mostly available at http://www.bibliotecamusica.it/cmbm/scripts/strumenti/scheda.asp?id=103&path=/cmbm/images/ripro/strumenti/vdm141/ Van der Meer described D♭♭ and G♭♭ which implies the Ptolemy-Vicentino enharmonic system. Van der Meer's interpretation assumed a 1/4 comma meantone tuning. Wraight, 2002, p. 122, Fig. 7, also showed this version of the keyboard range.

[93] Stembridge, 1993, pp. 57-58.

[94] Barbieri, 2008, p.314 (following Barbieri, 1983, p. 163) shows Vicentino's system as including D♭♭ and G♭♭. At 2008, p. 27 he indicates that 'If Vito Trasuntino had added three more 5ths at the bottom of the chain [of the 1601 harpsichord], he would have obtained the same harmonic range of Vicentino's *archicembalo*. This seems to be just what Vito did in his 31-keys per octave "Clavemusicum Omnitonum" (1606), the actual tuning of which is in any case hypothetical: see Fig. A.4.6 and Table A.4.3'.

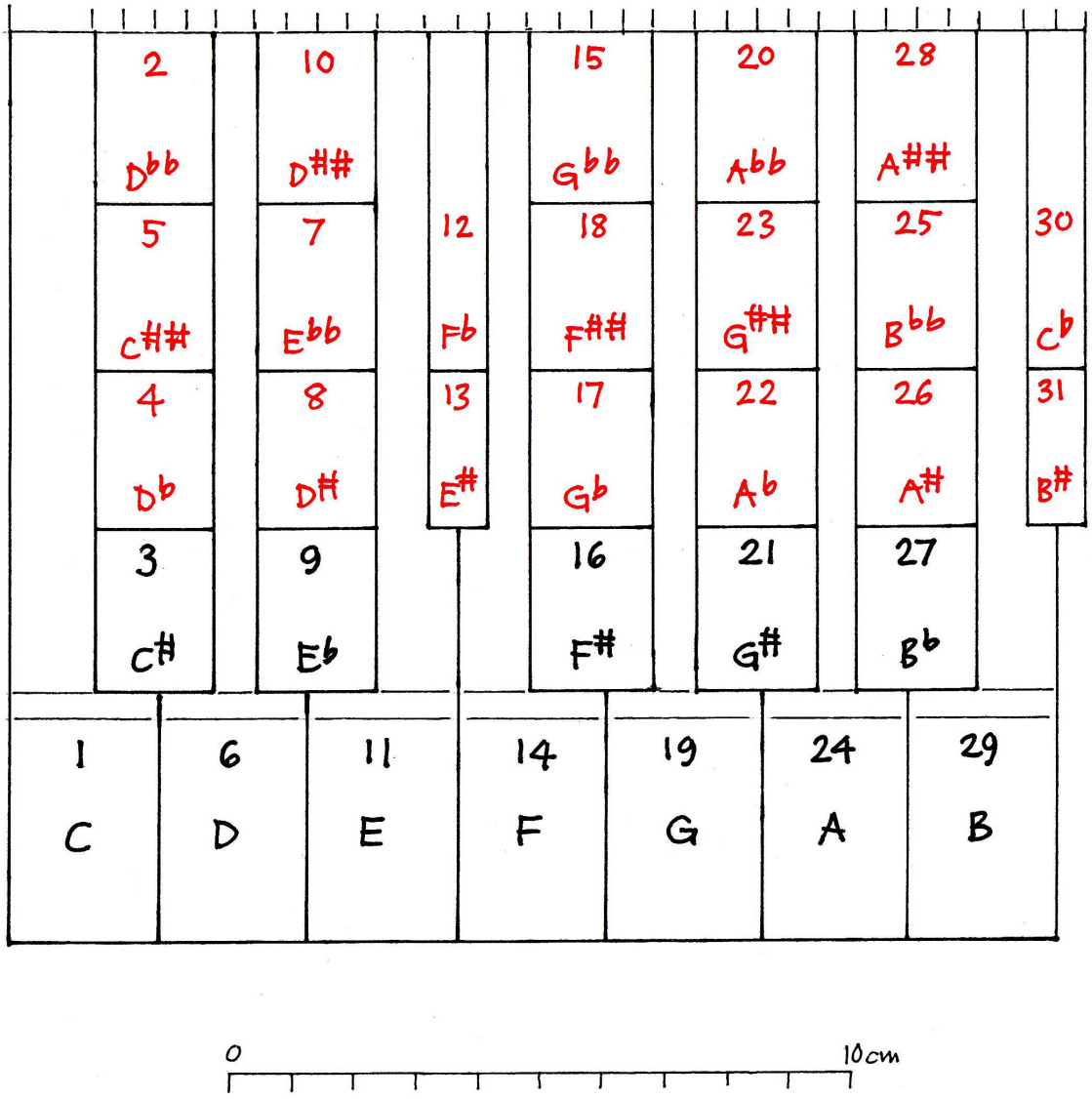


Fig. 11A. Trasuntino's 1606 Clavemusicum Omnitonum keyboard layout

15. For which enharmonic system was the Clavemusicum Omnitonum intended?

Whereas Zarlino added 5 ♯♯ notes to the 19-note *cimbalo cromatico* in order to produce his 24-note harpsichord, according to the interpretation of the range in Fig. 11 (G𝄫 to A♯♯) Trasuntino apparently added 5 𝄫 notes to his 1601 harpsichord for the design of the 1606 Clavemusicum Omnitonum. This completed Zarlino's scheme in a symmetrical fashion in the *ordine B quadro* and *ordine B molle*: there were now 12 notes in both the sharp and flat directions from the diatonic F - B. (See Table 5). There is no inherent gain with this symmetry in the *ordine* since there are two fewer tetrachords from ♯ notes with Zarlino's enharmonic. 38 Zarlino tetrachords were playable, of the possible 42, and without any mistuning using the 2/7 comma tuning.

It could seem at first sight as if the apparent omission of the 1601 harpsichord's E♯♯ and B♯♯ in the 1606 *keyboard* represents a retrograde step, but the fact is that these notes are not needed in the Ptolemy-Vicentino enharmonic in order to realise *ascending* tetrachords

from E♯ and B♯ since these require a different order of minor and major dieses. Could their omission from the *keyboard* could be the crucial indication that the Clavemusicum Omnitonum was conceived for the Ptolemy-Vicentino enharmonic, not for Zarlino's system?

E♯♯ and B♯♯ are *still* required in the Ptolemy-Vicentino enharmonic system, in *descending* tetrachords from F♯ and C♯ respectively (see Table 7/2), but these are replaced by G♭♭ and D♭♭ keys respectively in the 1606 31-note keyboard, according to the interpretation of Fig. 11. This causes a mistuning dependent on the actual temperament, or none, if a 31-note equal temperament system [ETS 31] is used.

Although the 1606 harpsichord according to Barbieri could have arrived at the same range as Vicentino's *Archicembalo* (G♭♭ to A♯♯), can we be sure that Trasuntino had abandoned Zarlino's approach?[95] It is the use of the 2/7 comma tuning for the 1606 monochord which creates the doubt about the utility of the Ptolemy-Vicentino enharmonic.

If we suppose that Trasuntino persisted with Zarlino's enharmonic conception, then the Clavemusicum Omnitonum with its 2/7 comma tuning and range of Vicentino's *Archicembalo* (G♭♭ to A♯♯) would have been incapable of *correctly* rendering ascending tetrachords from E♯ and B♯, since the E♯♯ and B♯♯ were lacking, although its predecessor, the 1601 harpsichord, could achieve this. This would indeed be curious for Zarlino's system since two of his notational examples of enharmonic tetrachords involve E♯ and B♯ (see Fig. 8, p. 26).[96]

What would be the consequence of supposing that Trasuntino retained the 1601 harpsichord's E♯♯ and B♯♯ thereby sacrificing the G♭♭ and D♭♭ respectively, but incorporated the three new ♭♭ notes? It would not change the total number of playable tetrachords; it would just exchange the correctly-tuned ascending tetrachords starting on F♭ and C♭ (Table 6/3 with G♭♭ and D♭♭) for the ones starting on E♯ and B♯ (Table 6/4 with E♯♯ and B♯♯). These latter two seem a more preferable choice for Zarlino's system than those on F♭ and C♭, and they also indicated by the monochord itself.

When we examine how the 31-note (Vicentino) keyboard range of Fig. 11 performs as a keyboard instrument in the Ptolemy-Vicentino enharmonic, then Table 7/2 shows that all 42 are realisable, but not with the monochord's 2/7 comma tuning. Although the ♭♭ notes in the 1606 harpsichord could be described as extending the "chromatic" range on the flat side, their *function* is to make a minor diesis (mD) step possible from *any* note, which is what is required by the Ptolemy-Vicentino enharmonic *tetrachords* in the descending F♯ - B♯ and ascending F – B. However, with 2/7 comma tuning, 14 of these tetrachords are mistuned, leaving only 28 tetrachords correctly playable (Table 7/1).

It will now be clear why we are not obliged to infer that Trasuntino was trying to follow Vicentino's work, as has been stated by some authors.[97] All Ancient Greek enharmonic tetrachords placed the smallest interval as the first step, which is what Trasuntino *could* have made possible with the Clavemusicum Omnitonum, in contrast to Zarlino's system,

[95] See note 94 for Barbieri's comment.
[96] Zarlino, 1558, p. 85 and p. 140.
[97] Hubbard, p. 31, "as late as 1606 a harpsichord was built to his [i.e. Vicentino's] prescription" and Russell, p. 32, "Trasuntino's example was inspired by the composer and theorist Nicola Vicentino...". Both statements can now be seen to be inadequately informed.

but the tuning of the monochord would then have been unsuited to the Ptolemy-Vicentino enharmonic. Thus, everything depends on what the monochord can tell us about the intended tuning of the Clavemusicum Omnitonum.

Nor need we infer that Vicentino's two-keyboard *Archicembalo* was the practical prototype for Trasuntino's Clavemusicum Omnitonum since the *development* of the designs is clearly different.[98] As we have seen, the evolution of this latter keyboard design is through the 1548 and 1601 instruments, not through a *cimbalo cromatico* with a second keyboard, which was Vicentino's route.

What now becomes clear is that Zarlino's enharmonic system is perfectly viable with the Clavemusicum Omnitonum and 2/7 comma tuning, but yielding only 38 correct tetrachords. If we wish to employ the Ptolemy-Vicentino enharmonic then the 2/7 comma tuning *has* to be replaced by another type. 31-note equal temperament (= 1/4.15 comma meantone, ETS 31 [31-note Equal Temperament System]) is the best replacement.[99]

Thus, the 1606 instrument supersedes Trasuntino's earlier variants, but even it it cannot realise all 42 tetrachordal possibilities with a 2/7 comma tuning. Trasuntino *appears* to have retained Zarlino's conception of the enharmonic tetrachords, although the next section shows that the monochord offers us further insights into this matter, since all turns on the tuning intended.

16. F♭ and C♭ in Trasuntino's 1601 harpsichord

We have concluded that Trasuntino's 1601 harpsichord was intended for Zarlino's enharmonic tetrachord system, and that the 1606 Clavemusicum Omnitonum could be tuned with Zarlino's 2/7 comma meantone for Zarlino's enharmonic, yielding 38 correctly-tuned tetrachords.

Could a 1601 monochord, or ruler division (if such a thing existed in the workshop), *already* have included flattened F♭ and C♭ notes (Section 8, p. 20), such as we find in the 1606 monochord? This idea might show us why the flattening in the 1606 monochord started at C♭ and not at G♭, or some other "flat note" in the circle of fifths (see Table 5. This question is pertinent because Trasuntino's 1601 harpsichord extended Zarlino's 1548 design with exactly these two notes on the flat side (*ordine B molle*).

The tuning difficulty for the 1548 harpsichord in 2/7 comma with F♭ and C♭ was explained in section 11 p. 28 and Fig. 9. Here we can examine the effect of the 1606 monochord on the actual tuning, and also gain an insight looking back to the 1601 instrument. The juxtaposition in Fig. 12 below shows the mistuning of intervals, ± in cents, due to the 2/7 comma temperament, referred to the *intended* size in Zarlino's enharmonic. Then in Fig. 13 we see the *actual* interval sizes resulting from the 1606 monochord frets.

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[98] The secondary literature expresses this simplistic view, such as in a current entry on Vicentino: "Only one keyboard instrument using his [Vicentino's] 31-note-to-the-octave system survives from the Renaissance: the 'Clavemusicum Omnitonum Modulis Diatonicis Cromaticis et Enearmonicis', built by Vito Trasuntino of Venice in 1606 to play the diatonic, chromatic and enharmonic."
https://en.wikipedia.org/wiki/Nicola_Vicentino (downloaded 13.12.2024)

[99] This implies a modification of 1/4 comma meantone with an extremely small widening of major thirds (+0.79 cents) and fifths (+ 0.19 cents).

Fig. 12. 1601 harpsichord with the 2/7 comma tuning (Zarlino's enharmonic system)

with 2/7 comma temperament ← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	-30 D♯♯	+30	F♭	F♭	F -30	E♯♯+50.3	A♯
G♭	B♭	-30 A♯♯	+30	C♭	C♭	C -30	B♯♯ +30	F♭
available notes ↑					↑available notes↑			
required notes ↓					↓ required notes ↓			
C♭	E♭	F♭♭		F♭	F♭	F	G♭♭	B♭♭
G♭	B♭	C♭♭		C♭	C♭	C	D♭♭	F♭

Fig. 13. 1601 harpsichord with the 1606 monochord's tuning[100]

Actual 1606 monochord tuning ← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	MIII 383.2	
C♭ 397.3	E♭	-26.7 D♯♯	69.2	F♭	F♭ 96.8	F -28.1	E♯♯ 373.4	B♭♭
G♭ 376.7	B♭	-29.3 A♯♯	73.4	C♭	C♭ 96.5	C -19	B♯♯ 393.7	F♭

Fig. 12 shows us how the *descending* tetrachords are distorted in Zarlino's 2/7 comma meantone, yet the *actual* 1606 monochord in Fig. 13 shows virtually correct sizes for these MD sizes. This is because the F♭ and C♭ notes are in reality closer to D♯♯♯ and A♯♯♯ (Table 1/A). The ascending tetrachord on F♭ in Fig. 12 is badly mistuned (+50.3 cents) for lack of a B♭♭.

Thus, seen harmonically, in the 1601 harpsichord (Fig. 4), the flattened C♭ is a mistake. One can easily see the effect of this in Table 5 where the C♭ - G♭ fifth is 26 cents too large, which is substantial. It is 5 times the tempering of a 1/4 comma meantone fifth, so virtually a "wolf" fifth and therefore unusable for its purpose.

However, in the *descending* tetrachord (Fig. 13), with the tuning of the Clavemusicum Omnitonum's monochord, an approximately correct MD (70.7 cents) has been achieved, because the C♭ is actually an A♯♯♯ (Table 1/A). Could this give an explanation of why the flattened notes were created?

17. A "correction strategy" for the Ptolemy-Vicentino enharmonic in the monochord?

Continuing the examination of the 1606 monochord, it is not obvious without some study of Zarlino's tuning that the effect of the flattening the ♭♭ notes (and some ♭ notes) is to compensate the inherent mistunings (30-cent quirks) in the minor and major diesis intervals (mD 50.3 cents and MD 70.7 cents) of tetrachords tuned in 2/7 comma meantone so that *nearly* normal (i.e. design) values result, in *some* places. Could this flattening of

[100] The notes are called E♯♯ and B♯♯ because they are closer to these in the monochord than G♭♭ and D♭♭.

notes have represented an intentional "correction strategy" for the Ptolemy-Vicentino enharmonic?

Regarding *Zarlino's* enharmonic, if Trasuntino had not made *any* changes to the 1606 monochord (with 2/7 comma tuning) there would only have been only 4 of the 30-cent quirks, assuming the range B♯♯ to A♭♭ (Table 5, 1606 monochord), and they would have been in the same place as in the 1601 harpsichord (Fig. 14).

Fig. 14. Mistuned tetrachords in the Clavemusicum Omnitonum
with Zarlino's enharmonic and the 2/7 comma tuning

← descending					ascending →			
383.2		50.3	70.7		70.7	50.3	383.2	
C♭	E♭	-30 D♯♯	+30 F♭		F♭	F +20.4	E♯♯ +30	B♭♭
G♭	B♭	-30 A♯♯	+30 C♭		C♭	C +20.4	B♯♯ +30	F♭

KEY
+ oversize intervals (cents)
- undersize intervals (cents)
enharmonic notes (Zarlino's definition)

However, if an hypothetical "correction strategy" were applied to the Clavemusicum Omnitonum's monochord for *Zarlino's* enharmonic system, then on balance there would be 2 improvements of the major diesis (MD, 70.7 cents, F♭ and C♭ descending) but 9 instances where it is *worsened*, (B♭ to G♭ descending, C to F descending, and C♭ to F♭ ascending: see Table 6/5). This is clearly such a poor outcome that it would not make any sense to correct for Zarlino's enharmonic with its 38 already correct tetrachords.

Turning to the Ptolemy-Vicentino enharmonic, if Trasuntino had not made any changes to the monochord, then Zarlino's 2/7 comma tuning would yield 28 correctly-formed tetrachords, but 14 with mistuning (Table 7/1). The reason is now not hard to understand: where there is a mix of flat and sharp notes in the ascending or descending tetrachords, this constitutes a mixing of the *ordine*, which (as we have already established in section 4) brings about a 30 cent quirk in the tuning.

However, if an hypothetical "correction strategy" were applied to the Clavemusicum Omnitonum's monochord with the Ptolemy-Vicentino enharmonic, i.e. with mD as the *first* interval in a tetrachord, then the 30 cent distortions from 2/7 comma tuning shown in Table 7/1 are modified, as now seen in Table 7/2. Trasuntino's "corrections" would bring the MD for F♭ to B♭ tetrachords (ascending) and F♯ to B♯ (descending) to within 10 cents of being correct (see Table 7/2). In addition the mD size is improved also to within roughly 10 cents for 7 of the 14 sizes, the results being better in the descending tetrachords.

The worsening of the monochord's actual tuning with Zarlino's enharmonic, but with some improvements in the Ptolemy-Vicentino system, *might* incline us to infer that Trasuntino had indeed abandoned the Zarlino enharmonic concept in the Clavemusicum Omnitonum, as was first suggested in section 15. In any event, "corrections" to the monochord would only make sense if the Ptolemy-Vicentino enharmonic is considered. However, this falls short of proof of a change to the Ptolemy-Vicentino enharmonic since our criteria of "improvement" and "worsening" assume that Trasuntino fully understood the implications of this hypothetical "correction strategy", which is not yet clear.

Although the descending tetrachords from F♯ to B♯ were all improved, the ascending tetrachords from B♭ to F♭ still had 5 significantly undersized mD intervals (B♭ to G♭), so the "correction" of all 28 intervals in these two groups was never achieved. When we examine why this should be then it becomes apparent that the problem lies in the fact the only "correction" we have is in the flattened ♭♭ notes, so that in fact they become closer to ♯♯♯ notes: this makes the descending tetrachords for F♯ to B♯ better, but the mDs of the ascending tetrachords are left mostly uncorrected. We do not have a comprehensive "correction strategy", merely a row of flattened ♭ notes from C♭ to G♭♭.

It is hard to imagine how a "correction strategy" could make any sense if it did not produce an improvement for the entire tuning. Even Zarlino's examples of the enharmonic (see Fig. 8, p. 26 above) were constructed with diatonic notes as the starting point, so there seems to be no good reason to suppose that the diatonic notes should be excepted from the "correction strategy".

Taking stock of the results of the "correction strategy" we can observe that the sizes on mD and MD are nearer being correct in the ascending flat tetrachords and descending sharp tetrachords, but that the ascending tetrachords from diatonic notes (which were correct) are now mistuned. The "correction" of some 23 intervals has been bought at the price of mistuning another 14.

The explanation that Trasuntino was attempting a "correction" of Zarlino's 2/7 comma meantone for the Ptolemy-Vicentino enharmonic can be discarded.

18. The 1606 monochord derived from the 1601 harpsichord through a mistake?

What has emerged from this discussion is that the flattened ♭♭ notes in the Clavemusicum Omnitonum's monochord have to be regarded as a mistake, if they are considered as part of a 2/7 comma tuning.

The possibility was considered in section 8 that the 1606 monochord was derived from an hypothetical monochord for the 1601 harpsichord, whereby the defective D♭♭ on the 1606 monochord would have been correct as the B♯♯ for the 1601 harpsichord. However, even this explanation seemed to imply that a mistake was made in flattening the F♭ and c♭ in the 1601 instrument.

From the outset of this enquiry it has seemed puzzling that a monochord from such an esteemed maker as Vito Trasuntino should have been defective. When we examine further what sort of operation could have led to the flattened F♭ and c♭ notes we find that the MIII / mIII confusion in constructing a monochord (discussed in section 8) is not the only possibility.

There is one simple construction which yields all these "errors": when the ♭♭ notes are constructed from the ♯♯ notes using the just major semitone interval (*semituono maggiore*) with the ratio 16:15 (111.73 cents). This is also a contravention of the rule established earlier that the ♭♭ notes should not be constructed from the ♯♯ notes, or vice versa.[101]

[101] Section 4, p. 13 above.

This is, of course, a mistake in the 2/7 comma tuning, but given the fact that there are elements of just intonation in the monochord, as discussed in section 4, it is perhaps not surprising.

Just intonation was the normal constructional scheme of the 16th-century harpsichord maker, and also of the organ builder, as Vincenzo Colombi's division revealed.[102] Not only is the 16:15 ratio is also to be found in Zarlino's just intonation "Tetrachordo Diatonico sintono di Tolomeo", nine pages preceding the diagram for the 2/7 comma tuning, but also in the diagram itself as the "*semituono maggiore*".[103] Zarlino later provided a comprehensive table of just intonation in his 1588 publication, reproduced here as Table 4B, which is a useful adjunct for understanding the octave structure.[104] Could all the labels on Zarlino's earlier, 1558 diagram (Table 4) for just intonation intervals have been naively misunderstood as requiring unmodified implementation, as discussed in section 4?

The correct size of the major semitone for 2/7 comma tuning cannot be seen in Zarlino's chart as a whole-number ratio because it cannot be expressed in this form. The correct size is the ratio 16:15 (111.73 cents) + 3/7 comma (= 120.95 cents, see Table 4A).[105]

However, the problem is yet more intricate: there is no simple relationship between a ♯♯ note and a ♭♭ note in Zarlino's 2/7 comma tuning. The size is actually 91.06 cents, but if the 16:15 ratio is used instead then an error of 20.67 cents results, which is sufficiently large that the fret positions would be visibly different. This difference in fret position would be -6.3 mm (flat) at D♭♭ on the 1606 monochord.

We start from the hypothesis that a ruler division from the 1601 instrument was kept in the workshop, which would have included B♯♯, E♯♯, F♭ and c♭, and then construct the *missing* ♭♭ notes for the 1606 monochord using a 16:15 construction from the ♯♯ notes. What we wish to understand is how well the 16:15 operation explains the *existing* frets in the monochord. We find that our newly-constructed ♭♭ notes correspond better to what we measure on the Clavemusicum Omnitonum's monochord than the position they should (theoretically) have.

Expressing these derivations with their errors in mm (Fig. 15 below), we see that 5 of the 9 constructions have errors of only between -0.1 and +0.8 mm when we compare the actual monochord (col. 4) with the result of the 16:15 derivation from (mostly) ♯♯ notes (col. 9). It was established in section 5, p.15 that -0.8 / +0.5 mm was the accuracy of working so this 16:15 construction hypothesis explains exactly what what we find in the 1606 monochord.

[102] Barbieri, 2002, p. 160, where the 16:15 ratio between b and c is shown in the numbered example as 192:180. See also note 50 above. The 16:15 ratio is found in Fogliano's monochord published in 1529, but known from a German manuscript of c. 1450 *Pro clavichordis faciendis* where a clavichord division is described. Through my dating of the watermarks of the paper we could speak of the clavichord instructions as "in or before 1464". See also Adkins, p. 240.
[103] For the "Tetrachordo Diatonico sintono di Tolomeo" see Zarlino, 1558, p. 122.
[104] Zarlino, Sopplimenti musicali, p. 155.
[105] See Zarlino's diagram, 1558, p. 130 (not numbered). It is the interval at e - f, a - b♭, and b - c.

Fig. 15. Defective ♭ & ♭♭ notes in the 1606 monochord
showing their derivation through the 16:15 ratio from a ♯♯ or ♭♭ note

1	2	3	4	5	6	7	8	9
note	key no.	2/7 MT mm theoretical	actual monochord mm	error mm	♯♯ note	2/7 MT mm theoretical	16:15 ♯♯ >♭♭ mm	16:15 derivation mm
C	32	0	**533.2**	0				
D♭♭	33	517.9	526.5	-8.6	B♯♯	527.0	519.4	+7.1
E♭♭	38	463.7	466.3	-2.6			465.6	+0.7
F♭	43	415.1	419.4	-4.3			416.2	+3.2
G♭♭	46	387.1	390.2	-3.1	E♯♯	393.8	389.5	+0.7
A♭♭	51	346.5	350.0	-3.5			350.1	- 0.1
B♭♭	56	310.2	314.5	-4.3			313.2	+1.3
c♭	61	277.7	281.4	-3.7			280.6	+0.8
d♭♭	64	259.0	263.1	-4.1	b♯♯	263.5	261.5	+1.6
e♭♭	69	231.8	234.5	-2.7			234.7	- 0.2

Key to Fig. 15:
Columns 1-3 show the note name, key number and monochord, with the correct mm value for Zarlino's 2/7 comma (MT = meantone) tuning. Col. 3 is taken from Table 3C, col. 5, with the corrected C fret position (533.2 mm).
Column 4 shows the actual 1606 monochord fret positions expressed in mm (relative to C).
Column 5 gives the relationship of the actual monochord to the *correct* size (col. 3) The results are from from Table 3C, col. 7, shown in mm. All the notes are flat.

Columns 6-7 are as per columns 1 and 3, but for the extra ♯♯ notes considered.
Column 8 shows the monochord fret position (expressed in mm, as per col. 4) which results from taking the actual monochord ♯♯ position and using the 16:15 ratio to derive the ♭ or ♭♭ position.
Column 9 compares col. 8 with col. 4, i.e. the result of the 16:15 operation is compared with the actual monochord so that we see the deviation of the actual result from the hypothetical 16:15 operation. If the 16:15 derived fret is higher pitched than the **monochord** (col. 4) then the result in col. 9 is designated +.

Less convincing for the hypothesis are the errors for B♭♭ and d♭♭ (+1.3 and +1.6 mm respectively), but even these deviations are in keeping with the errors listed in Table 3G for the preferred interpretation of the monochord's tuning as 2/7 comma, some of which, D♯ and E♯, were also as large.

The large error (+7.1 mm) in column 9 for fret 33 D♭♭ disappears if we understand the note as a B♯♯ (cols. 6 and 7), which could have been taken, unchanged, from the 1601 division; then fret 33 (526.5 mm) is merely +0.5 mm high of its correct value (compare col. 4 with col. 7).

Even the incorrect c♭ can be explained with the 16:15 operation, this time descending in pitch from b♯♯ (an hypothetical size constructed as the octave from fret 33) with an error of only +0.8 mm.

A difficulty which remains is that the F♭ string would be too long by 3.2 mm, if constructed with a 16:15 ratio from an E♯♯ at fret 42 (390.2 mm, labelled G♭♭ in Table 1). However, it is

remarkable that this flattened F♭ on the monochord is actually correct (i.e. 419.4 mm, only +0.7 mm sharp) if constructed with the 16:15 operation from a *correct* E♯♯ (393.8 mm).[106] This fact shows us that the 1606 monochord could have been derived from the 1601 division, which included correctly-formed B♯♯ and E♯♯ notes; at least these notes were used in 1601, according to Pesenti's report.

Despite this slight uncertainty about the error with the E♯♯ it seems that we have indeed found the method used to construct the ♭♭ notes on the 1606 monochord, and in addition, it was probably (but it is not proven) already implemented in a predecessor made in 1601.

The part of the puzzle which remains is why the B♯♯ note should have been correctly constructed, but the F♭ and c♭ notes were not. It seems that firstly the ♯♯ series was completed accurately from A♯♯.[107]

The next stage, the construction of the c♭ and F♭ notes, reveals a change of method with 16:15 ratios used in descending fashion from the ♯♯ notes, instead of minor semitones from the c and F notes, which would be correct for 2/7 comma.

Why this change of method was implemented, and whether it was already used for the 1601 harpsichord, remains to be investigated. However, a plausible explanation can now be given how the 1606 monochord could have been derived from the 1601 harpsichord division, *including* its correct B♯♯ and E♯♯ notes, but with F♭ and c♭ notes constructed using the ratio 16:15 (111.73 cents. Further ♭♭ notes were added by the same 16:15 ratio operation from the remaining ♯♯ notes. In this way the incorrectly-flattened ♭♭ notes for Zarlino's 2/7 comma tuning were constructed for the 1606 monochord.

19. Three solutions for the monochord's discrepant frets

Three solutions have been considered to explain the defective frets in the 1606 monochord:

1. That the apparent ♭♭ notes of the monochord were actually *intended* to be ♯♯ and ♯♯♯ notes with a range G♭ - A♯♯♯. This ♯♯♯ hypothesis would have no useful purpose in *Zarlino's* enharmonic scheme. In the Ptolemy-Vicentino enharmonic scheme, although all the descending tetrachords starting on F♯ - B♯ would be correct, all the 7 ascending tetrachords on the flat side, F♭ - B♭, would still be subject to the 30 cent quirks for 2/7 comma tuning. Thus, having ♯♯♯ notes in the monochord would not provide an adequate explanation of the apparently flattened ♭♭ notes. (Section 8, and result in Section 14)

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[106] From Table 3C, a correct E♯♯ = 393.8 mm so F♭ = x25/24 = 420.1 mm. The F♭ fret = 419.4 mm on the monochord.

[107] The E♯♯ and B♯♯ could have been constructed in 1601 without error (by the ascending MIII then descending mIII technique) from E♯ and B♯ respectively, which yields the 25:24 minor semitone. Alternatively, they could have been constructed directly from the 25:24 ratio. Although it might initially seem as if such large number ratios would be too unwieldy, the history of monochord divison indicates that there were simplifications which could be used. Instead of dividing the length for E♯ into 25 parts it suffices to divide it into 5. Then, the fifth part at E is further divided into 5, and of these 1 part taken in the direction of the tuning pins; that is the 24th part. See also note 31.

2. That the 1606 monochord was laid out with flattened notes from c♭ through the ♭♭ notes as a "correction strategy", in order to compensate for the distortion of interval sizes which result from Zarlino's 2/7 comma tuning in the Ptolemy-Vicentino enharmonic scheme. This is ineffective and could only have been attempted by Trasuntino if he did not fully understand what he was doing. (Section 17)

3. That the 1606 monochord was probably copied from the 1601 instrument design with its E♯♯ and B♯♯. The flattened F♭ and c♭ notes, and the ♭♭ notes were added by the operation of a 16:15 ratio, from the ♯♯ notes. In this way the incorrectly-flattened ♭♭ notes for Zarlino's 2/7 comma tuning were constructed on the 1606 monochord. (Section 18)

It is this third explanation which explains the facts, since it shows a simple method by which the ♭♭ notes on the monochord were constructed and why they are consistently of the wrong size for Zarlino's 2/7 comma tuning.

20. Why was the 16:15 ratio used to construct the ♭♭ notes?

Some remarks made by Pesenti on the tuning of Zarlino's and Trasuntino's harpsichords have a bearing on the origin of the 16:15 construction which brought about the incorrectly flattened ♭♭ notes for the 2/7 comma tuning in the 1606 monochord.

Pesenti was employed by Nicolò de Rossi in Venice between 1621-1634 to tune and play the 1601 Trasuntino harpsichord, which was then in Rossi's possession.[108] According to Pesenti's account the instrument had not been tuned since Vito's death (aged 86, in October 1612, so for some 8 years), allegedly because Rossi could not find anyone who would undertake it. In this way Pesenti tells us that he had to discover the pitches of the various notes for himself, not that he was equipped with a description of the notes, or even a monochord. Pesenti even stated that the tuning of instruments was not his trade [*professione*]. Whether this was to exonerate himself from any possible mistake, or to magnify the achievement of decyphering the tuning of the harpsichord is not clear.

Only in 1642 did Pesenti come across Zarlino's harpsichord, made for the theoretician in 1548 by Dominico Pesarese, the split notes of which the blind musician compared with Trasuntino's 1601 harpsichord in his 1645 publication. Rasch inferred that the tuning of Zarlino's harpsichord could have been conveyed to him through the instructions written on

[108] Pesenti, Introduction, "A PROFESSORI DI MVSICA, Per Maggior Intelligenza". Stembridge, 1993, pp. 16-18, gives the original Italian and an English translation as far as the tuning is concerned. Rasch, pp. 77-81 gives the entire introduction and an English translation. The section on the tuning of the instruments is as follows: "Prima nell'A, La, mi, re vi e il B. molle, il # & il # maggiore, & in quel di Vido e anco l'Istesso; In quel del Zerlino vi e il b, fa, b, mi, b, molle, & il #, ma vi e questa differenza, che in quel di Vido vi e un # maggiore di piu. Al c, sol, fa, ut del Zerlino vi e il #, & il # maggiore. In quel di Vido vi e il b, molle, il #, & il # maggiore, In D,La,sol,Re del Zerlino, vi e il b molle il # & il #, maggiore, & in quel di Vido vi e l'istesso. In E, La, mi del Zerlino vi e il b molle, & il #, In quel di Vido vi e il b molle il #, & il # maggiore. Nel F,Fa,ut del Zerlino vi e il # & il # maggiore in quel di Vido vi e il b molle, il # & il # maggiore. In G,sol,Re,ut del Zerlino vi e il b molle, il #, & il # maggiore & anco in quel di Vido e L'istesso. Di maniera che nascono in quel di Vido doi corde Fra il b,Fa,b,mi & il, c,sol, Fa ut, cioe il # maggiore, di b,fa,b,mi, & il b, molle di c,sol,Fa,ut, che e semituono maggiore. Nascono anco in quel di Vido doi corde tra E.La ,mi, & F,Fa,ut il # maggiore d'E,La,mi, & il b molle di F,Fa,ut, semituono parimente maggiore."

the back of the nameboard by Zarlino himself, which had been reported by Burney in 1770.[109] Unfortunately for us Pesenti did not describe Zarlino's tuning.

According to an observation by Rasch we can infer from Pesenti's remarks that Trasuntino's harpsichord "was to be tuned" in 1/4 comma meantone since the interval C♭-B♯♯ is referred to as being a major semitone (*semituono maggiore*).[110] Rasch presumes here that Pesenti intended to tell us the *absolute size* of the interval, not that it was the larger of the two semitones in a tempered tuning. This presumption must be examined because even Zarlino was imprecise in the description of his harpsichord, when he referred to the *semituono maggiore* being divided into two parts.[111] However, when describing earlier the distribution of the comma in his tuning he was accurate in mentioning the increase of the *semituono maggiore* by 3/7 of a comma.[112]

In 16th-century theory the *semituono maggiore* is defined by the ratio 16:15 (111.73 cents), but it does not occur in 1/4 comma meantone; here the major semitone cannot be defined by a whole number ratio, although it is close in size: 117.11 cents. Thus, there is a 5.38 cents difference between the 1/4 comma meantone major semitone and the *semituono maggiore*. (Zarlino's 2/7 comma major semitone is yet wider at 120.95 cents.)

There is though the aspect of the "quirk" in a tuning which has already been examined and thereby now makes this discussion easier. In a 31-note tuning with 1/4 comma meantone the interval at C♭ - B♯♯ is subject to a 6.07 cent quirk. As a result the 1/4 comma meantone major semitone is, for certain intervals, theoretically 117.11 - 6.07 = 111.04 cents, merely 0.69 cents smaller that the *semituono maggiore*.[113]

This interesting coincidence shows that the 1/4 comma meantone major semitone is almost identical with the 16:15 interval size, but only for the intervals in Trasuntino's harpsichord formed between ♭ and ♯♯ notes, i.e. between F♭ - E♯♯, and C♭ - B♯♯. Other 3-step sizes, (in a 31-note tuning with 1/4 comma meantone = major semitone), have the normal 117.11 cent size. Pesenti might not have known all this and calculation before the invention of logarithms would have been tedious.

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[109] Rasch, p. 48, note 20. According to Burney, p. 253, "At Florence I found the harpsichord of Zarlino, which is mentioned in the second part of his Harmonical Institutions, p. 140 [i.e. 1st edition]. This instrument was invented by Zarlino, in order to give the temperament and modulation of the three *genera*, the diatonic, chromatic, and enharmonic; and was constructed under his direction in the year 1548, by Dominico Pesarese: it is now in the possession of Signora Moncini [Mancini; Barbieri, 2008, p.25], widow of the late composer Piscetti [*recte* Pescetti; Barbieri, 2008, p. 25]. I copied Zarlino's instructions for tuning it, from his own hand-writing, on the back of the fore-board; but I shall reserve them, and the particular description of this curious instrument, for the History of Music, to which they more properly belong."

[110] Rasch, p. 51: "That Trasuntino's instrument was to be tuned in meantone temperament can be derived from a remark by Pesenti that is puzzling at first sight. After having described Trasuntino's instrument he says that there are two keys between B and C, namely B♯♯ and C♭, that make up a major, that is, a diatonic semitone. Strictly speaking the interval B♯♯ - C♭ is not a diatonic semitone. But if it is taken as three steps (C♭-B♯-C-B♯♯) in a 31-note octave it is a diatonic semitone in a 31-note tuning, and therefore in meantone tuning." This is not clear; see note 114.

[111] Zarlino, 1558, p. 140: "...il quale fece Maeſtro Dominico Peſareſe fabricatore eccellente di ſimili ſtrumenti, nel quale non ſolamente li semituoni maggiori ſono diuſi in due parti,..."

[112] Zarlino, 1558, p. 132: " In tal maniera adunque haueremo vna terza chorda, la qualeſegnaremo con la lettera C, & ſarà la ſeconda del primo Tetrachordo, che con la E contenerà il Semituono maggiore, accreſciuto di tre ſettime parti."

[113] The audible difference from a 0.69 cent difference in intervals would produce a beat of 0.13 Hz at f¹ (pitch a¹ = 415 Hz), which in practice would be unrecognisable in the background of the unsteady tones of a harpsichord.

Of course Pesenti could have investigated the size of the interval empirically, after tuning the harpsichord: if he had re-tuned F♭ as a pure fifth *below* C♭ and then re-tuned the F♭ an octave higher as a pure major third *above* B♯♯ he could have found that the F♭ - F♭ octave was *virtually* pure, as would be the case when C♭ - B♯♯ is a major semitone with the ratio 16:15, as in just intonation from B - C. That could have led him to describe the interval C♭ - B♯♯ as a *semituono maggiore*.

There will, of course, be a tempered 31-note tuning where the major semitone at C♭ - B♯♯ is *exactly* equal to a *semituono maggiore*, after allowance for the quirk, but it is not ETS 31 (1/4.15 comma). The required tuning is a 1/4.02 comma meantone ("near 1/4 comma") where the fifths are only 0.022 cents larger than in 1/4 comma meantone, a difference which is so small that in practice a 31-note tuning in 1/4 comma meantone would probably produce an apparent *semituono maggiore* between F♭ - E♯♯, and C♭ - B♯♯.[114]

It would not be surprising if Pesenti had tuned Trasuntino's 1601 harpsichord in 1/4 comma meantone since the temperament was well known and easy to set, but on Pesenti's account this tuning is his own finding, not information supplied with Trasuntino's harpsichord.[115] It is inherently unlikely that Trasuntino left instructions for tuning the harpsichord since he could command a fee for doing this work. In pre-enlightenment times it was unusual for workers to impart the information with which they earned their livelihoods.

Nevertheless it is interesting for our understanding of why the 16:15 ratio operation was constructed in the monochord. If Pesenti thought that there could be a *semituono maggiore* between C♭ - B♯♯, why should a monochord maker in 1606 not have thought it possible to construct the C♭ from the B♯♯, the F♭ from E♯♯, or the A♭♭ from G♯♯ with a 16:15 ratio? This is exactly the procedure that was used, as was found in the 1606 monochord for the Clavemusicum Omnitonum.

We can now see how two parts of the puzzle overlap. Firstly, Zarlino's harpsichord was designed for 2/7 comma tuning and the range was chosen, ending at G♭ in the *ordine B molle* side, in order that no tuning quirks would occur in tetrachords.

Secondly, Trasuntino's 1601 harpsichord exceeded Zarlino's usable range with 2/7 comma tuning so he needed a solution: either more keys or a different tuning.

Trasuntino would have needed 35 notes in order for his F♭ - B♯♯ range to render the tetrachords correctly with 2/7 comma tuning. The additional notes would have been the ♭♭ notes on B, E, A D, G, C, and F. (see Table 5, col. 5, "1601", Tables 6/2 and 6/2A)

1/4 comma, or something with slightly less tempered fifths ("near-1/4 comma"), produces a *better* tuning for the new notes he incorporated. In this way the tetrachords from F♭ and C♭

[114] This is probably what Rasch, p. 51, had in mind when he wrote "... in a 31-note octave it is a diatonic semitone in a 31-note tuning, and therefore in meantone tuning." These expressions can only be approximate. In 1/4.02 comma meantone 31 fifths are smaller than 18 octaves by 5.379 cents, the amount which reduces a major semitone to a 16:15 ratio *semituono maggiore*.

[115] In his expression "Trasuntino's harpsichord *was to be tuned* in meantone temperament" [my emphasis] Rasch implies a prescription, at least in Pesenti's mind, but perhaps as if there were instructions from Trasuntino how the instrument should be tuned. That would go a little further than Pesenti's account permits. We could imagine that a monochord existed, but then we would have to infer that the harpsichord tuners in Venice were so intimidated by the device, or the harpsichord, that they declined to attempt a tuning of Rossi's instrument between 1612 and 1621.

would have become usable, although not exactly correct. Such a tuning would be characterised by a *semituono maggiore* at C♭ - B♯♯, or something so close to it that the tuner would not have detected the difference.

The near-1/4 comma tuning is not required in order that scales employing C♭ and F♭ be correctly tuned; it is only needed for the correct rendering of *tetrachords*. This simple, but significant fact indicates that Trasuntino probably adopted something like a 1/4 comma meantone in 1601 because it resolved the D♯♯ / F♭♭, A♯♯ / C♭♭, B♯♯ / D♭♭ and E♯♯ / G♭♭ incompatibilities, thereby making all tetrachords possible on C♭ and F♭ in Zarlino's enharmonic system.

This explanation implies that in 1601 Trasuntino clearly understood Zarlino's enharmonic system and its limitations with the 2/7 comma tuning. In the *fully* enharmonic, 31-note 1606 Clavemusicum Omnitonum, this tuning close to 1/4 comma meantone would have been required in order to enable all 42-tetrachords, which we might be tempted to see as the *completion* of Zarlino's enharmonic programme.

That this supposed "completion" was achieved without the 2/7 comma tuning would not have been tolerated by Zarlino since this was an integral part of his enharmonic system with the MD - mD - MIII order of intervals in the tetrachords. It seems probable that Zarlino first devised the 2/7 comma tuning from just intonation, then constructed the tetrachords to fit the tuning. As we have seen (Section 10), had Zarlino used the Ptolemy-Vicentino order of intervals in the tetrachord, then he would have needed 31 notes to enable the tetrachords already playable with his 1548 harpsichord. These would not even have been the same 31 notes as in the Clavemusicum Omnitonum: they would have included the familiar E♯♯ and B♯♯, but also required ♯♯♯ notes on F, C. G, D and A. (see Table 7/1A)

Thus, Zarlino's major diesis has a pivotal position in both the tuning and the tetrachord. Trasuntino's solution for the 31-note keyboard with a new tuning was not the *completion* of Zarlino's scheme, but an abandonment thereof.

The "change of method" for making the monochord, discussed in section 18, whereby the F♭ and c♭ were constructed not with the 25:24 minor semitone, but with a 16:5 ratio from ♯♯ notes, can now be explained: it is not a mistake in 2/7 comma tuning, but evidence of the application of a *different* tuning, that of something close to 1/4 comma meantone. This 16:15 ratio cannot determine exactly what tuning Trasuntino intended since the three possibilities, 1/4 comma meantone, "near 1/4 comma meantone" and ETS 31 are so close in size that they can only be be distinguished theoretically.

The 16:15 operation demonstrates a probable point of transition in the workshop practice from the 1591 chromatic harpsichord, or Zarlino's specification, to the 1601 instrument. That there was a ruler division in the workshop (if not actually a monochord) for the 1601 harpsichord was suggested by the analysis in section 18, but it also fits the development of the tuning, as far as we can trace it.

When we consider the *practical* matter of how a monochord might be extended in 1/4 comma meantone to include ♭ or ♭♭ notes, there is no whole number ratio with which one can construct a major or minor semitone. Neither is it easy to construct tempered fourths and fifths, thus it is clearly more practical to use a 16:15 ratio from the "sharp side". Even though it is theoretically incorrect in just intonation to mix the *ordine*, in a monochord intended for something close to 1/4 comma meantone it would yield the right magnitude of

result. In fact, this 16:15 procedure would give worst-case results only 0.73 cents flat, or 0.26mm flat in a theoretically-correct 1/4 comma tuning, which is less than the manufacturing error in the monochord (established as -0.8mm / +0.5mm).

As has been found, the basic tuning system of the monochord is 2/7 comma. The 16:15 operation used *within* this system would yield a fret at D𝄫 (worst case) which would be 20.67 cents flat, or 6.26 mm in error. This is why we find such large discrepancies in the 𝄫 notes in the monochord.[116]

The only difficulty which remains is to explain how a new tuning close to 1/4 comma meantone could have been combined with Zarlino's 2/7 comma tuning in the *same* monochord. This is a nonsense, which somehow came about, perhaps through a lack of clear understanding in the workshop on the part of whoever made the monochord, or through a lack of supervision on the part of Vito.

However, this conflation of two tuning schemes in the monochord has the advantage for us in showing the development of Trasuntino's instruments from the Zarlino 2/7 comma tuning before 1601, to a more or less equal tuning for 31 notes when the range exceeded that of Zarlino's tetrachordal system in 1601 and 1606. This is what the monochord indicates and which Pesenti's account alone cannot reveal.

21. Was the Clavemusicum Omnitonum's range G𝄫 - A𝄪 or A𝄫 - B𝄪?

Now that it has been found that the monochord was initially based on the 2/7 comma tuning system, but that the 𝄫 notes were constructed later using a feature of 1/4 comma meantone only found in a 31-note tuning, the question remains what the range of the Clavemusicum Omnitonum was intended to be: G𝄫 - A𝄪 - or A𝄫 - B𝄪? In other words, was the keyboard range identical with Vicentino's *Archicembalo* G𝄫 - A𝄪 (col. 6, Table 5) or did it include E𝄪 and B𝄪 notes? (col. 7, Table 5). The whole matter hinges on how to interpret the E𝄪 and B𝄪 notes in the monochord.

The conventional view of a 31-note keyboard, whether Vicentino's or Trasuntino's, has been that through the five-fold division of the tone there are 𝄪 and 𝄫 notes. It is as if one took Zarlino's 1548 harpsichord with a four-fold division of the tone, which provided 𝄪 notes, and then extended it on the flat side with 𝄫 notes. As we have seen, the development in Trasuntino's workshop was not from a 1548-type design to the 1606 instrument, but included (at least) the intermediate 1601 harpsichord.

If we had not known anything about the 1601 instrument we would naturally have inferred from the decyphered monochord that the Clavemusicum Omnitonum was equipped with E𝄪 and B𝄪 notes. It is only the possibility, discussed in section 18, that the monochord contains an earlier division from the 1601 harpsichord which creates doubt about this inference. Could the 1606 monochord simply be a lazy and thoughtless continuation of an earlier practice?

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[116] Comparing column 4 with column 8 in Table 1, the 𝄫 notes appear a little better tuned if considered as part of a 1/4 comma meantone, rather than a 2/7 comma tuning, but they are still not well tuned. The five 𝄫 notes from D𝄫 to B𝄫 range from -5 to -22.1 cents, with an arithmetic mean of 13.1 cents. This does not mean that the monochord was constructed for 1/4 comma meantone. It merely reflects the fact that 𝄫 notes are flatter in 1/4 comma meantone than in 2/7 comma, with the result that they *appear* to be better tuned.

For sure, the B♯♯, E♯♯, F♭ and c♭ were required in the 1601 design, as was seen in section 18. What cannot be discerned from constructional considerations of the monochord is whether the frets at the E♯♯ and B♯♯ notes (frets 46 and 33 respectively) were thought of as ♯♯, or whether this was perhaps only a lazy copying of a pre-existing design?

The 16:15 fret-construction operation in Trasuntino's monochord shows that Vito had found a suitable solution for the tuning of an instrument incorporating flat notes from C♭ and beyond, on the flat side (*ordine B molle*). The monochord division itself suggests that this change of direction in the tuning had already taken place in 1601. Thus, the B♯♯, E♯♯, F♭ and c♭ could have been taken *knowingly* from the 1601 design.

However, the keyboard layout of the Clavemusicum Omnitonum (Photo 3 and Fig. 11A) *appears* to indicate that E♯♯ and B♯♯ had been omitted, because the key pattern no longer corresponds to the 1601 harpsichord scheme (Fig. 10A). As will now be shown, this appearance is not decisive.

The schematic drawings of the three keyboards presented here for the 1548, 1601, and 1606 instruments (Figs. 5A, 10A, and 11A respectively) were drawn full size, using the same keyboard width, and using dimensions for the 1548 and 1601 harpsichords, which are representative of their maker's practice. This makes the keyboard layouts directly comparable.[117]

In this way a simple, but significant detail of construction becomes apparent: the more "split sharps" are aligned behind each other, the wider the sharps (the part presed by the player) must be. This is for the obvious reason that the keylevers for the front sharps have to pass *under* the other sharps. Thus, each drawing includes the nominal keylever width division projecting *behind* the keycovers.

Stembridge touched on the problem when he asked why the 1601 harpsichord should have the E♯♯ and B♯♯ keys placed *between* E and F, and B and C (respectively)?[118] The anomaly is (for example) that E♯♯ is higher in pitch than F, but appears in a lower position on the keyboard (Fig. 10A).

What else could Trasuntino have done in 1601? He could have placed the E♯♯ at the same position of key 15 on the 1606 harpsichord (Fig. 11A, in the back row, behind F♯). This would then have required keycovers as long as the Clavemusicum Omnitonum (147 mm) although only two keys (E♯♯ and B♯♯) would have been required in the "back row". This is a rather cumbersome solution.

When we approach the problem from the 1606 harpsichord's keyboard we can see that he could have retained the three split sharps from the 1601 instrument (see nos. 10, 11 and 12 in Fig. 10A, 1601 instrument, p. 31), but would have been obliged because of the limited space to have only three split sharps behind the F (16 F♯, 17 G♭, and 18 F♯♯ in Fig. 11A, 1606 instrument, p.34). This would have required the three sharps (C♯ - D♭♭ - C♯♯ and F♯ - G♭ - F♯♯) in the 1606 keyboard to be narrower (15 mm) than the others (20 mm). This would also have been an odd arangement.

[117] This was not so for my earlier publication, Wraight, 2002, where all split sharps have the same width.
[118] Stembridge, 1993, p. 54.

The option which was *not* available to him in the 1606 keyboard was to have 20 mm wide banks for *all* sharps AND three split sharps between E and F; there was simply too little space for that. This is what the drawing of the keylever width *behind* the keycovers makes clear for us. It is a purely practical matter, decided by the minimum possible width of keylevers and the space available.[119]

Thus, we cannot infer from the keylever position (no. 15 in Fig. 11A, 1606 instrument) that it must have been a G♭♭. It might well have been thought of as an E♯♯, which is what the monochord would indicate. Seen in this way we realise that Stembridge's probing question shows that it is only certain ingrained ideas concerning the Clavemusicum Omnitonum's tuning possibilites, which incline us to assume that a double raised note in the back rank behind the F key must be a G♭♭.

The only infelicity for the player if key 15 is an E♯♯ is that the spacing of the keys for C♯♯ - E♯♯ is now wider than for the comparable C♯ - E♯. However, the raised key would be positioned correctly vis-a-vis its pitch.[120]

The information gleaned from this monochord indicates the previously unknown result that Trasuntino used Zarlino's tuning and enharmonic system until his 1601 harpsichord, at which point he had to modify it. However he persisted with those elements of Zarlino's approach until 1606, which gave a priority to the ♯♯ side. Although the keyboard appears, according to the conventional interpretation, to incorporate Vicentino's harmonic range of G♭♭ - A♯♯, the monochord testifies that Trasuntino retained the E♯♯ and B♯♯ notes of the 1601 harpsichord. Trasuntino's Clavemusicum Omnitonum was not based on Vicentino's *Archicembalo*, but on Zarlino's theoretical system, and then developed from it. Of course, through the near 1/4 comma tuning, E♯♯ and G♭♭, B♯♯ and D♭♭ would have been functionally interchangeable.[121]

22. The historical background of the Clavemusicum Omnitonum

The possibility that this 1606 instrument left the workshop with an *apparently* defective monochord initially led me to question whether an instrument maker of Vito Trasuntino's reputation, at the age of 80, just six years before his death, was still closely involved in the final stages of manufacture. Of course, whether Trasuntino was responsible for this monochord, or his workers, we will not be able to determine, but the nameboard of the harpsichord does bear his confident statement: VITO DE TRASUNTINIS VENETO AVCTORE. A letter found by Iain Fenlon in the Novellara archives, the only one known in Vito's hand, confirms that Trasuntino was active at this time, having written to Camillo in March 1607 about the *Clavicimbano*.[122] Pesenti's testimony implies that Trasuntino maintained the 1601 harpsichord until his death in 1612, and was therefore active in his capacity as an instrument maker. We must at least conclude that if he had any oversight of

[119] The keyboard width of the Clavemusicum Omnitonum was based on 24" Venetian (= 695.5 mm), Slightly wider keyboards (25") were occasionally used for the same range by Dominico Pesarese and Celestini. See Wraight, 2011, pp. 93-94. The wider, 25" range would not materially alter the possibilities of including the extra notes because the extra 1" (29 mm) is spread over 29 naturals.

[120] Stembridge, 1993, p. 58, discusses the difficulty he saw as a player in manoeuvering between the ♯♯ and ♭♭ notes.

[121] Stembridge, 1993, p. 57 assumed this interchangeability in his discussion, albeit without examining the tuning.

[122] I am grateful to Iain Fenlon for this communication and a copy of the letter. That it is the only one known in Vito's hand is due to Marco Di Pasquale (private communication). See Appendix.

construction in the workshop it was not sufficient to prevent the curious conflation of tuning systems made with the monochord. None of this bodes well for the final assessment of Vito's involvement in the matter of the monochord as a defective tuning device.

Vito Trasuntino would surely have been aware of Vincenzo Colombi's work in building an *Arciorgano* for Vicentino in 1561, since both instrument makers worked in Venice. We may therefore infer that Trasuntino would likely have been informed about Vicentino's enharmonic system, even without reading the *L'Antica Musica*.

Nevertheless Trasuntino, for reasons we cannot determine, took Zarlino's published route with the 2/7 comma tuning and its idiosyncratic enharmonic system with a leading Major Diesis as the first interval.

Camillo Gonzaga was 25 when the Clavemusicum Omnitonum was made for him, but he had succeeded to the title in Novellara in 1596. The Novellara court, through the *maestro de cappella* Giaches [Jacques] De Wert (until his death in 1596), had strong musical connections with the Este court in Ferrara where Vicentino had been employed and where one of his *Archicembali* was used by Luzzaschi, who died in 1607, the year the Clavemusicum was delivered.[123] Luzzaschi could therefore have been an advisor in the matter of an enharmonic instrument, but since he had the reputation of being able to play an *arciorgano* with fluency and wrote compositions for it, he probably contributed to the contemporary interest in chromatic music, not only in Ferrara.[124] Bottrigari gave an insight into the difficulty of maintaining and playing such an instrument with two keyboards, so Vito's harpsichord with a single keyboard may have been a practical response, conceivably instigated by Luzzaschi, to the difficulties of constructing a playable instrument.[125] It is not implied that any of these musicians were interested in performing Vicentino's enharmonic compositions, since by all accounts such performances stopped when the peripatetic musician died around 1576.

There was, however, already a previous workshop connection to the Ferrara court since Alessandro Trasuntino had supplied a harpsichord in 1547 for Ercole II, on which the 21-year old Vito probably worked.[126] Another harpsichord, made by Vito after Alessandro's death, was supplied to Alfonso II, possibly just after 1559, and contained additional d♯ and a♭ notes.[127]

Despite Trasuntino's intended A♭♭ - B♯♯ range, the Clavemusicum Omnitonum was probably tuned to a nearly-equal 31-note tuning, which enables the same functionality as the Vicentino range G♭♭ - A♯♯. The monochord would have been of no practical assistance

[123] The last secure date of Vicentino's activity in Ferrara is 1555. See Kaufmann p. 33.
[124] Palisca, 2006, p. 92 and note 54 quoting Pietro Cerone, El Melopeo, f. 1041 correctly as his source, but Cerone appears merely to have repeated Bottrigari's information, pp. 40-41. Bottigari states that the Archicembalo contained 26 diatonic notes and 130 strings, which does not agree with the design Vicentino published: this had 27 diatonic notes (in each keyboard) and 132 strings in total.
[125] Bottrigari, p. 41
[126] Attributed in Wraight, 1997, Part 2, pp. 294-296, W54 "Bortolus" (a faked inscription). The attribution to Alessandro is confirmed by Montanari's publication, pp. 190-193 and 221-223, of a 1698 Florentine inventory. This harpsichord is now in the Tagliavini Collection, Bologna. Vito Frassonio was born in Monastier near Treviso in 1526 and was not related to Alessandro, who died in 1552. Vito married in Venice, in 1543; see Appendix. It appears that Vito took the Trasuntino surname in order to carry on Alessandro's business. See Di Pasquale 2019/1.
[127] Now in Schloß Köpenick, Berlin. See Wraight, 1997, Part 2, pp. 334-335. W366 in my catalogue. After our joint examination of the harpsichord in 1982, Martin-Christian Schmidt (b.1946-d.2000) independently supposed a connection to the Trasuntino workshop (personal communication and unpublished restoration report, 1986-1989, p. 43).

in the tuning, as Tiella already determined, but it reveals for us some of the complicated story of the instrument's intended range.[128]

23. A suitable tuning for any enharmonic instrument

Tunings for keyboard instruments are usually only considered for the chromatic genus. How they fare in the enharmonic genus is not obvious without a closer study.

In the descending tetrachord on F♭ shown in Fig. 16 below, the "design cents" line shows how little difference there is between D♯♯ and E♭ (20 cents) with Zarlino's tuning in a tetrachord for which it was designed: one might well miss hearing such a step in a piece of music.

Fig. 16.

Zarlino's enharmonic ← descending Zarlino 2/7 comma (= 0.286 comma)			
M III	mD	MD	
383.24	50.28	70.67	design cents
C♭	E♭ -30c	D♯♯ +30	F♭
383.24	20.39	100.56	actual cents

An example from Vicentino's *Musica prisca caput,* of which Wild created a performance, depends specifically on the effect of the soprano voice moving upward in the enharmonic genus by mD and MD steps from B♭♭ to D.[129] There are two occasions in this piece (B♭ to A♯♯ and D♭♭ to C♯) when Zarlino's 2/7 comma tuning would provide only a 20 cent difference (half that of 1/4 comma meantone), but this is actually perfectly normal for Zarlino's tuning because the step from ♭ to ♯♯ (or ♯ to ♭♭) is always 20 cents (see Table 8A).

Thus, there are two different matters which arise with the 2/7 comma tuning:

1. In some compositions involving the enharmonic, Zarlino's tuning would only supply a rather small mD interval, of only 20 cents.
2. In the tetrachords, occasional ± 30 cent quirk intervals can occur with Zarlino's tuning, as we see in the "actual cents" line in Fig. 16 (above).

This view of the sizes of intervals in the enharmonic has attracted little attention in our time so that the problem appears to be virtually unknown. Lindley even suggested a 0.3 comma tuning for Vicentino's *Archicembalo* (equivalent in this example to the Clavemusicum Omnitonum) which is a shade worse than Zarlino's 2/7 comma (= 0.286 comma) and would result in quirk sizes of ±39.4 cents.[130] Expressed in the form we have seen until now:

[128] Tiella, 1975, p. 141.
[129] The magnitude of such minor dieses can be appreciated by listening to Wild's example of the soprano singing alone: https://mtosmt.org/issues/mto.14.20.2/wild_examples.php?id=13&nonav=true (The pitch of the voices was adjusted to be correct with software.) This performance is recommended to the reader.
[130] Lindley, 1982, pp. 389-390. The tuning is defined as 0.3 comma meantone, which places it between 1/3 and 2/7 comma.

Fig. 17.

Lindley 0.3 comma Zarlino's enharmonic		← descending	
M III	mD	MD	
382.01	53.96	68.52	design cents
C♭	E♭ -39c	D♯♯ +39	F♭
G♭	B♭ -39c	A♯♯ +39	C♭
382.01	14.56	107.93	actual cents

This shows the distortion of the steps from an exceptionally large MD to a virtually non-existent mD with only 14.56 cents ("actual cents"). This would probably be imperceptible in performance, so it has hardly any use in the enharmonic.[131]

By comparison, and as a means of illustrating a suitable solution, here is 1/4 comma meantone presented in the same format as in Fig. 18, which has only ±6.07 cent quirks (see also Table 8B).

Fig. 18.

1/4 comma tuning Zarlino's enharmonic		← descending	
M III	mD	MD	
386.31	41.06	76.05	design cents
C♭	E♭ -6c	D♯♯ +6	F♭
G♭	B♭ -6c	A♯♯ +6	C♭
386.31	34.99	82.12	actual cents

Even when 6.07 cent quirks occur in the system, 1/4 comma meantone results in minimal distortions and ensures a good contrast between MD and mD.[132]

1/4 comma meantone is also conceptually well adapted to Zarlino's enharmonic (and just intonation) since it contains two whole-number ratios, which are also found in Zarlino's enharmonic tetrachord: 128:125 and 5:4.[133] Of course, this only becomes evident when one considers an instrument with more than 12 notes in the octave since the minor diesis 128:125 ratio is formed between the adjacent ♭ and ♯ , or ♯ and ♭ notes. In his 2/7 comma tuning Zarlino took a different course in retaining the 25:24 chromatic semitone size unaltered, and adapted thirds and fifths in his tuning to fit it. As already shown, Zarlino appears to have taken little interest in the enharmonic for compositions; his 1548 harpsichord with its limited range could avoid the difficulties shown above.

As early as the 17th century it was recognised by Lemme Rossi, and then independently by Christian Huygens, that 1/4 comma meantone applied to a 31-note tuning gave an almost closed circle of fifths, missing the target by merely 6 cents.[134] However, as the current analysis shows, discussions of tunings for enharmonic instruments which

[131] Lindley's awareness of this issue with his tuning was expressed thus: "it would render Vicentino's thirty-one dieses somewhat unequal." Lindley, 1982, p. 389.
[132] 1/4 meantone cent values are given to two decimal places in Table 3.
[133] There are also other whole number intervals in the octave division for 1/4 comma meantone: 25:16 = 772.63 cents (G♯), and 8:5 = 813.69 cents (A♭).
[134] Barbieri, 2008, p. 330. Lemme Rossi published his analysis in 1666, pp. 86-88. Barbieri, 2008, p. 332, discusses Huygens' *Novus Cyclus harmonicus* [31] published in 1691. Tanaka, p. 75, working from reported information, suggested that the Clavemusicum Omnitonum was probably tuned with a 31-note equal tuning, in which he was apparently following Huygens' suggestion concerning Vicentino's Archicembalo; see p. 68.

concentrate on the narrowing of the fifths and adjustment of the thirds deal with only a part of the matter of achieving a suitable temperament; the size of the enharmonic minor diesis must also be suitable.

The tetrachordal analysis has been used as a theoretical tool in this article in order to show the limitations of the 2/7 comma tuning, but these Greek tetrachords were far removed from musical practice at this time. Viewed harmonically (in just intonation), a ♭♭ note will not be called upon in counterpoint to combine with a single ♯ or ♯♯ note. Thus, the tetrachords represent situations which do not occur in normal counterpoint and the 30 cent distortions described here might in practice have had little significance. However, the fact that Zarlino skillfully avoided exposing the tuning limitations of his 1548 harpsichord shows that he probably used a tetrachordal analysis in his construction and understanding of the tuning system. As has been seen in section 20, Trasuntino's change of tuning system only has a sense if one tests the tetrachordal possibilities.

Despite the difficulties with the size of the mD which have been shown above, it is possible to render all of the tetrachords with Zarlino's system correctly, even when using the 2/7 comma tuning; it just requires a larger compass than 28 notes of the 1601 harpsichord. As Table 6/2A shows, 35 notes would be required in order to incorporate correctly the F♭ and c♭ notes, but it would be extremely difficult to implement this in a single keyboard. Trasuntino's 31-note keyboard is a small masterpiece in the art of instrument making.

If the Ptolemy-Vicentino enharmonic system is used then the requirement changes. At the bottom of Table 7/1A we see that 45 notes would be required in the Clavemusicum in order to render these tetrachords correctly using 2/7 comma meantone, but ♯♯♯ and ♭♭♭ notes are clearly impractical in addition to the ♯♯ and ♭♭ notes, for reasons of space in a single keyboard.

24. Tuning Zarlino's and Trasuntino's instruments

We should see Zarlino's 2/7 comma meantone tuning as the correct and intended tuning for the 1548 harpsichord; in this way the 30 cent discrepancies discussed above do not occur: Zarlino had skillfully coordinated his harpsichord range to fit his tuning system.

Benedetti published a tuning method for 2/7 comma meantone in 1585, which now has greater significance since it is clear that the Trasuntino workshop was using Zarlino's temperament for a monochord division.[135] That Trasuntino's instruments before 1601 were probably intended for 2/7 comma meantone also shows that Zarlino's publications had a practical effect in Venice beyond the circle of musicians.

Although the 2/7 comma meantone *could* be laid out accurately on a monochord, it is not as easy to tune as 1/4 comma meantone, which has just thirds. Lindley described the results of his own long search for an accurate method of tuning the temperament.[136] He also described Benedetti's method.[137] The inevitable inaccuracies of even an accurately

[135] Benedetti, pp. 281-282.
[136] Lindley, 1997, pp. 187-190.
[137] Lindley, 1987, pp. 160-161 (in German) and Lindley, 1990, p. 19, Fig. 8 which explains Benedetti's procedure starting from G with a minor sixth to E♭, then by a chain of narrowed fifths B♭ - F – C – G – D – A – E – B - F♯ - C♯ to G♯, leaving the wolf fifth at g♯ to e♭. It seems strange that Benedetti should have chosen G as the starting note since this does not correspond to the lowest note of an Italian harpsichord,

made monochord would not have been able to reveal the subtleties of the tuning, so that a musician would have had to rely on his own judgement to set a correct temperament. It has also been established that the flawed monochord with elements of two tuning systems would never have been able to set a correct temperament for the Clavemusicum Omnitonum, but the "Trecta Cordo" can nevertheless be seen as providing significant testimony of the development of instruments and their tuning in the Trasuntino workshop.

It is possible to tune the Clavemusicum Omnitonum in Zarlino's 2/7 comma meantone. The only disadvantage is (as shown in Table 6/3) that merely 38 of the possible 42 tetrachords would be correctly tuned.

If it is desired to render all 42 tetrachords correctly with a 31-note keyboard, then the 2/7 comma tuning is inadequate. The decyphered monochord shows that Trasuntino probably recognised this difficulty and sought a remedy by abandoning Zarlino's enharmonic scheme and using another tuning. Three tunings were distinguished in section 20:

1/4 comma meantone (major semitone [MS] = 117.11 cents)
1/4.02 comma meantone ("near-1/4 comma"; MS = 111.73 cents = *semituono maggiore*)
1/4.15 comma (ETS 31, Equal temperament System, 31 notes; MS = 116.13 cents)

In practice, these three systems might each yield a major semitone between between F♭ - E♯♯ and C♭ - B♯♯ would could appear in practice to be a 16:15 *semituono maggiore*, at least on superficial examination. Thus, the telling detail of the *semituono maggiore* mentioned by Pesenti does not give us adequate purchase on the problem. The almost microscopic differences in the distribution of the syntonic comma (1/4 - 1/4.02 - 1/4.15) are also inadequate to give us an idea of the magnitudes we are dealing with in these tunings, and this has led to the idea that the differences are negligible.[138] However, in enharmonic tetrachords a skillful tuner would be able to recognise whether (for example) in the ascending tetrachord from F♭ the major third from the G♭♭ - B♭♭ had a beat rate equivalent to other major thirds. In this way apparently negligible tuning differences for the chromatic genus become evident in the enharmonic.

In 1/4 comma meantone an E♯♯ (instead of G♭♭ in this tetrachord from F♭) would produce a major third of 392.4 cents instead of the required 386.3; this is result of the 6.1 cent quirk. Even the 1/4.02 comma tuning ("near-1/4 comma" tuning), which yielded the *exact* 16:15 ratio *semituono maggiore*, would show a 5.4 cent difference compared with normal major thirds.[139] Both of these tunings would yield audible differences in the size of the major thirds for even a moderately skilled tuner. Only with ETS 31 would all major thirds have the same size: 387.1 cents, barely wider than a pure third (386.3 cents).

Thus, the conclusion is unavoidable that if the tuner of the Clavemusicum Omnitonum wishes to ensure that the major thirds in all tetrachords have the same size, he *must* use an ETS 31 temperament. Given that the entire logic for Trasuntino's move away from Zarlno's 2/7 comma tuning was based on the correct rendering of tetrachords, it is fair to infer that if he had any skill as a tuner he would have tuned in this way with ETS 31. It might not have been clear to Trasuntino, or a contemporary tuner, that he was creating a

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which was usually F or C. It might be explained through his procedure, which essentially tunes the range between e♭ and g♯: the e ♭ is more conveniently reached from a G.

[138] As opined by Barbieri, 2008, p. 315.

[139] In 1/4 comma meantone 5.4 cents is also the difference between a tempered and a pure fifth.

temperament much different from what was considered to be "1/4 comma meantone" in his day.

Although in our time "1/4 comma meantone" has come to imply a tuning with beatless major thirds, as Lindley has observed, Aaron's instructions, which are often cited as a source of this temperament, do not clearly specify beatless thirds; the instructions can be read as allowing a slight beating in the C - E major third, and actually describe "impure" [*spuntate*] thirds.[140] Lanfranco is even clearer in specifying a regular tuning where the major thirds are slightly widened.[141]

The major third of ETS 31 is 387.1 cents (386.31 cents is pure), so it requires only that a "very slow wave" (0.15 Hz at e^1, at a pitch of a^1 = 415 Hz) be set in the major third c^1 - e^1 for the correct setting of a distant third (e.g. E𝄪 / G𝄫 - B𝄫). Thus, 16th-century tuners might often unwittingly have produced a tuning which was effectively ETS 31, but since they were only tuning 12-note (or perhaps 14-note) per octave instruments, this would never have been evident to them.

An equal temperament system is not the only means of correctly rendering the 42 tetrachords we have been considering, which effectively form the enharmonic system we can impute to Zarlino. A 35-note keyboard would have been sufficient with Zarlino's 2/7 comma tuning for Zarlino's enharmonic, but using the same tuning, 45 notes would have been necessary for Vicentino's enharmonic. If the task is to render the 42 tetrachords correctly with a 31-note keyboard, then ETS 31 is necessary, regardless of the enharmonic system envisaged.

It cannot be confirmed that 1/4 comma meantone was the temperament *intended* for the Clavemusicum Omnitonum. In using ETS 31 Trasuntino could be seen as returning, in effect, to an established tradition of tuning after a period of pursuing enharmonic experiments with Zarlino's 2/7 comma meantone.

[140] See Lindley, 1974, pp. 139-144 (which includes a translation of Aaron's instructions into English). Aaron states that the C-C octave should be "well unified" [bene unita], but he requires only that the C - E third be "sonorous and correct" [sonore e giusta], adding, "as far as possible" [cioe unita al suo possibile]. Later he writes of "impure" [spuntate] thirds, which are not compatible with 1/4 comma meantone. For a discussion of the translation of *spuntate* see Wraight, 2024, 'Vicentino's enharmonic lute...', pp. 2-3. My discussion in Wraight, 1997, Part 2, pp. 129-132 had not yet solved the problem of translation. "Aaron" is in the 1523 edition; "Aron" in the 1529 and 1539 editions.

[141] Lanfranco, pp. 132-134 and Lindley, 1974, p. 145 with an English translation of the relevent phrase "Terza maggiore va alzata in modo chel senso piu non ne voglia - Third be raised in such a way that the ear wishes no more".

Conclusions

The monochord supplied by Trasuntino's workshop as a tuning aid for the 1606 Clavemusicum Omnitonum was found by visual and numerical investigation to have been closest in design to a 2/7 comma tuning. The tuning of the frets could be clearly distinguished from 1/4 comma meantone. (Sections 2-3)

The design of the monochord was based on Zarlino's diagram (*Le Istitutioni Harmoniche*, 1558, p. 130), with the same c^1-e^2 range, but the diatonic notes were naively transferred from the diagram without the required tempering. (Sections 4-6)

Zarlino developed a 2/7 comma tuning and an enharmonic monochord based on retaining the just intonation chromatic semitone (ratio 25:24, 70.67 cents), but narrowing the major thirds and the fifths. In a 31-note system this leads to a gap of 30 cents between nominally equivalent pitches at opposite ends of the spiral of fifths (e.g. A♯♯ and C♭♭). (Section 3)

Zarlino's and Vicentino's enharmonic systems differ in the order of the dieses. Zarlino's system had the sequence: Major diesis - minor diesis - Major third, which is atypical of known tetrachordal systems, but was the result of his priority to develop a tuning compatible with the diatonic, chromatic, and enharmonic genus. Vicentino placed the minor diesis first (as did Ptolemy), before the Major diesis, which leads to a different requirement in the notes of the keyboard. (Section 10)

Zarlino's enharmonic harpsichord, made for him in 1548 by Dominico Pesarese, with 24 notes per octave (not 19 notes as implied by the drawing), ensured the correct tuning of tetrachords within the limitations of his tuning system. This explains the limited range of the instrument as G♭ - A♯♯; additional notes on the flat side (C♭ and F♭), as required by Zarlino's own instructions, would have created incorrectly-tuned tetrachords (±30 cents), and were avoided. (Sections 9 and 11)

Zarlino's unusual enharmonic system also exhibits an economy in the number of notes required, and practicality in the keyboard construction. 31 notes per octave (including five ♯♯♯ notes) would have been required in the Ptolemy-Vicentino enharmonic system in order to play the same 24 tetrachords as Zarlino's 1548 instrument could achieve. (Section 10)

The ♭♭ frets in the monochord are too flat and present the major puzzle of this study. They are close to being ♯♯♯ notes, but this interpretation would make no sense in Zarlino's enharmonic system so the explanation was rejected. (Section 14)

Neither can the ♭♭ notes can be explained as an attempt to correct Zarlino's 2/7 comma tuning for Vicentino's tetrachordal system. (Section 17)

It was found that the ♭♭ frets were constructed from the ♯♯ notes on the monochord using the ratio 16:15 (*semituono maggiore* [major semitone] in just intonation), as was probably intended for a previous harpsichord, such as the 1601 Trasuntino with a 28-note range. (Section 18)

This 16:15 ratio has no place in Zarlino's 2/7 comma tuning, but it is exactly the size produced by a near-1/4 comma tuning (1/4.02 comma), which would provide some correction of the major thirds in the tetrachords, albeit not a perfect adjustment. Since Trasuntino's 1601 harpsichord extended Zarlino's 1548 harpsichord range on the "flat"

side with F♭ and C♭ a tuning other than 2/7 comma would have been required in order to avoid extra keys. This necessity in the tetrachord tuning, and the probability that it was incorporated in a monochord division before 1606, gives us the confidence to infer that Trasuntino had understood the enharmonic tuning requirements of a wider-range instrument than Zarlino's 1548 harpsichord. (Section 20)

The tuning of the monochord indicates that Trasuntino, coming from the Zarlino tuning and its 1548 harpsichord, probably thought of the range of the Clavemusicum Omnitonum as being A♭♭ - B♯♯ and not the G♭♭ - A♯♯ usually described. The use of a near-1/4 comma meantone tuning would have made the B♯♯ and E♯♯ notes functionally the same as D♭♭ and G♭♭. (Section 21)

Although apparently intended as a tuning aid, the monochord (even without the ♭♭ notes) was not made accurately enough to be used as such. Furthermore the incorrect use of some just intonation frets, and the incompatible ♭♭ notes from a different tuning system, rendered the monochord inherently incapable of setting the temperament of the Clavemusicum Omnitonum. (Section 21)

Vito Trasuntino was still actively involved in the workshop in 1606, as the only letter in his hand, found by Iain Fenlon, testifies. It is therefore surprising and remains unexplained how such an incoherent monochord with two tuning systems could have been produced. (Section 22)

Despite this inadequacy, the monochord provides significant testimony which has enabled the reconstruction of the development of the tuning of chromatic and enharmonic instruments in Trasuntino's workshop. The Clavemusicum Omnitonum should not be considered to be the same as, or following, Vicentino's *Archicembalo* since the lines of development were different. (Section 22)

The logic of Trasuntino's alterations to the monochord reveals his orientation on the tetrachords as decisive for a suitable tuning, which eliminates 1/4 comma meantone as unsuitable. (Section 24)

ETS 31 [31-note Equal Temperament System] was the only tuning which would have satisfied the requirement that *all* major thirds in the tetrachords be similarly tuned. Since it is only marginally different from 1/4 comma meantone in 12-note per octave instruments it would probably not have been perceived by practitioners of tuning as a different temperament. In effect, Trasuntino had returned to an established tuning tradition after his experiments with Zarlino's 2/7 comma tuning. (Section 24)

Table 1 Monochord lengths, cents, Zarlino's 2/7 comma and 1/4 comma tuning

	no.	2/7 MT cents	- (22)	+ (18)	FH cents	FH 2015 mm	- (31)	+ (9)	1/4 MT cents
C	32	0			0	532.3			
D𝄫	33	50.28	31.3		19.0	526.5	22.1		41.1
C♯	34■	70.67		8.9	79.6	508.4		3.5	76.1
D♭	35	120.95		4.7	125.6	495.1		8.5	117.1
C𝄪	36	141.34		13.0	154.3	486.9		2.2	152.1
D	37■	191.62		10.7	202.3	473.6		9.1	193.2
E𝄫	38	241.90	12.7		229.2	466.3	5.0		234.2
D♯	39	262.29		2.9	265.2	456.7	4.0		269.2
E♭	40■	312.57		5.7	318.7	442.8		8.4	310.3
D𝄪	41	332.96		10.5	343.5	436.5	1.8		345.3
E	42■	383.24		2.5	385.7	426.0	0.6		386.3
F♭	43	433.52	20.8		412.7	419.4	14.7		427.4
E♯	44	453.91		3.1	457.0	408.8	5.4		462.4
F	45■	504.19		5.3	509.5	396.6		6.1	503.4
G𝄫	46	554.47	16.9		537.6	390.2	6.9		544.5
F♯	47■	574.86		6.3	581.2	380.5		1.7	579.5
G♭	48	625.14	1.0		624.1	371.2		3.6	620.5
F𝄪	49	645.53		6.8	652.3	365.2	3.2		655.5
G	50■	695.81		3.6	699.4	355.4	0.2		696.6
A𝄫	51	746.09	20.2		725.9	350.0	11.7		737.6
G♯	52■	766.48	1.6		764.9	342.2	7.7		772.6
A♭	53	816.76	3.2		813.6	332.7	0.1		813.7
G𝄪	54	837.15		0.1	837.2	328.2	11.4		848.7
A	55■	887.43	3.1		884.3	319.4	5.4		889.7
B𝄫	56	937.71	26.7		911.0	314.5	19.8		930.8
A♯	57	958.1	9.8		948.3	307.8	17.5		965.8
B♭	58■	1008.38	7.6		1000.8	298.6	6.0		1006.8
A𝄪	59	1028.77		1.3	1030.1	293.6	11.7		1041.8
B	60■	1079.05		1.3	1080.3	285.2	2.6		1082.9
c♭	61	1129.33	25.8		1103.5	281.4	20.5		1124.0
B♯	62	1149.72	1.3		1148.4	274.2	10.5		1158.9
c	63■	1200	2.9		1197.1	266.6	2.9		1200.0
d𝄫	64	1250.28	30.3		1220.0	263.1	21.1		1241.1
c♯	65■	1270.67	6.3		1264.6	256.4	11.5		1276.1
d♭	66	1320.95	11.2		1309.8	249.8	7.3		1317.1
c𝄪	67	1341.34		0.6	1341.9	245.2	10.2		1352.1
d	68■	1391.62		4.1	1395.7	237.7		2.5	1393.2
e𝄫	69	1441.90	22.7		1419.2	234.5	15.0		1434.2
d♯	70	1462.29		0.2	1462.5	228.7	6.7		1469.2
e♭	71■	1512.57	11.8		1500.8	223.7	9.5		1510.3
d𝄪	72	1532.96	3.3		1529.7	220.0	15.5		1545.3
e	73■	1583.24	1.0		1582.4	213.4	3.9		1586.3

Key to Table 1

Column 1. Name of note.

Column 2. The number on the keyboard and monochord. The black square shows the position of black (ebony) fillets in the slots. see Photo 4.

Column 3. The cent value of the note in Zarlino's 2/7 comma tuning (MT = meantone, since this tuning is often referred to as "2/7 comma meantone"). This was calculated to 3 decimal places and rounded. The position of the cent value in the column reflects the position of the key in the keyboard, for easier visual orientation (cf. Photos 3 and 7).

Columns 4 and 5. - (blue) and + (red) values showing the deviation of the monochord fret from the theoretically correct 2/7 comma tuning (col. 3).

Column 6. "FH" = Friedemann Hellwig. The ratio of the length of a string at a fret compared with the C length (fret 32), expressed as a cent value. (These show minor differences, less than 1 cent, compared with Hellwig's data, due to rounding differences or slight errors in Hellwig's calculations).

Column 7. The lengths given by Hellwig 2015 in mm for the monochord frets. N.B. D♭ was given as 497.1 mm by Hellwig, which does not accord with his own cent calculation; this appears to be a typing error. The correct value has been taken as 495.1 mm (col. 7), which does match his cent calculation.

Columns 8 and 9. - and + values showing the deviation of the monochord fret from the theoretically correct 1/4 comma tuning (col. 10)

Column 10. 1/4 comma meantone (MT = meantone) in cents.

Table 1A. Monochord lengths, cent values, and Zarlino's 2/7 comma tuning
with additional ♯♯, ♭♭, and ♯♯♯ notes

NOTE	key no.	key no.	2/7 MT cents	- (22)	+ (18)	note	2/7 MT cents	-	+	mono. cents	FH 2015 mm
C	32	1	0							0	532.3
D♭♭	33	2	50.28	31.3		B♯♯	20.39	1.4		19.0	526.5
C♯	34■	3	70.67		8.9					79.6	508.4
D♭	35	4	120.95		4.7					125.6	495.1
C♯♯	36	5	141.34		13.0					154.3	486.9
D	37■	6	191.62		10.7					202.3	473.6
E♭♭	38	7	241.90	12.7		C♯♯♯	212.01		16.9	292.2	466.3
D♯	39	8	262.29		2.9					265.2	456.7
E♭	40■	9	312.57		5.7					318.7	442.8
D♯♯	41	10	332.96		10.5	F♭♭	362.85			343.5	436.5
E	42■	11	383.24		2.5					385.7	426.0
F♭	43	12	433.52	20.8		D♯♯♯	403.63		9.1	412.7	419.4
E♯	44	13	453.91		3.1					457.0	408.8
F	45■	14	504.19		5.3					509.5	396.6
G♭♭	46	15	554.47	16.9		E♯♯	524.58		3	537.6	390.2
F♯	47■	16	574.86		6.3					581.2	380.5
G♭	48	17	625.14	1.0						624.1	371.2
F♯♯	49	18	645.53		6.8					652.3	365.2
G	50■	19	695.81		3.6					699.4	355.4
A♭♭	51	20	746.09	20.2		F♯♯♯	716.2		9.7	725.9	350.0
G♯	52■	21	766.48	1.6						764.9	342.2
A♭	53	22	816.76	3.2						813.6	332.7
G♯♯	54	23	837.15		0.1					837.2	328.2
A	55■	24	887.43	3.1						884.3	319.4
B♭♭	56	25	937.71	26.7		G♯♯♯	907.82		3.2	911.0	314.5
A♯	57	26	958.1	9.8						948.3	307.8
B♭	58■	27	1008.38	7.6						1000.8	298.6
A♯♯	59	28	1028.77		1.3	C♭♭	1058.66			1030.1	293.6
B	60■	29	1079.05		1.3					1080.3	285.2
c♭	61	30	1129.33	25.8		A♯♯♯	1099.44		4.1	1103.5	281.4
B♯	62	31	1149.72	1.3						1148.4	274.2
c	63■		1200	2.9						1197.1	266.6
d♭♭	64		1250.28	30.3		b♯♯	1220.39	0.4		1220.0	263.1
c♯	65■		1270.67	6.3						1264.6	256.4
d♭	66		1320.95	11.2						1309.8	249.8
c♯♯	67		1341.34		0.6					1341.9	245.2
d	68■		1391.62		4.1					1395.7	237.7
e♭♭	69		1441.90	22.7		c♯♯♯	1412.01		7.2	1419.2	234.5
d♯	70		1462.29		0.2					1462.5	228.7
e♭	71■		1512.57	11.8						1500.8	223.7
d♯♯	72		1532.96	3.3						1529.7	220.0
e	73■		1583.24	0.8						1582.4	213.4

Key to Table 1A (and schematically for others as well)

Column 1. Name of note

Column 2. The number on the keyboard and monochord

Column 3. The key number in Photo 2 and Fig. 11A, p. 34. Discussed in section 21.

Column 4. The cent value of the note in Zarlino's 2/7 comma tuning. This was calculated to 3 decimal places and rounded. The position of the cent value in the column reflects the position of the key in the keyboard, for easier visual orientation (cf. Photos 3 and 7)

Columns 5 and 6. - (blue) and + (red) values showing the deviation of the monochord fret from the theoretically correct 2/7 comma tuning (col. 3)

Column 7. Additional notes discussed in the text

Column 8. Cent value of col. 6 in the 2/7 comma tuning

Columns 9 and 10. - and + values showing the deviation of the monochord fret from the theoretically correct 2/7 comma tuning (col. 7

Column 11. "mono." = monochord. The ratio of the length of a string at a fret compared with the C length (fret 32), expressed as a cent value. (These show minor differences, less than 1 cent, compared with Hellwig's data, due to rounding differences or slight errors in Hellwig's calculations)

Column 12. The lengths given by Hellwig 2015 in mm for the monochord frets. N.B. D♭ was given as 497.1 mm by Hellwig, which does not accord with his own cent calculation; this appears to be a typing error. The correct value has been taken as 495.1 mm (col. 7), which does match his cent calculation.

Table 2. Size of major thirds (M III)

	no.	2/7 MT cents	- (22)	+ (18)	FH cents	FH 2015 mm	M III -	M III +
C	32	0			0	532.3	385.7	
D𝄫	33	50.28	31.3		19.0	526.5		393.7
C♯	34■	70.67		8.9	79.6	508.4	377.4	
D♭	35	120.95		4.7	125.6	495.1	383.9	
C𝄪	36	141.34		13.0	154.3	486.9	383.3	
D	37■	191.62		10.7	202.3	473.6	378.9	
E𝄫	38	241.90	12.7		229.2	466.3		394.9
D♯	39	262.29		2.9	265.2	456.7		387.1
E♭	40■	312.57		5.7	318.7	442.8	380.7	
D𝄪	41	332.96		10.5	343.5	436.5	382.4	
E	42■	383.24		2.5	385.7	426.0	379.2	
F♭	43	433.52	20.8		412.7	419.4		400.9
E♯	44	453.91		3.1	457.0	408.8	380.2	
F	45■	504.19		5.3	509.5	396.6	374.8	
G𝄫	46	554.47	16.9		537.6	390.2	373.4	
F♯	47■	574.86		6.3	581.2	380.5	367.1	
G♭	48	625.14	1.0		624.1	371.2	376.7	
F𝄪	49	645.53		6.8	652.3	365.2	377.8	
G	50■	695.81		3.6	699.4	355.4	380.9	
A𝄫	51	746.09	20.2		725.9	350.0	377.6	
G♯	52■	766.48	1.6		764.9	342.2	383.5	
A♭	53	816.76	3.2		813.6	332.7	383.5	
G𝄪	54	837.15		0.1	837.2	328.2	382.8	
A	55■	887.43	3.1		884.3	319.4	380.3	
B𝄫	56	937.71	26.7		911.0	314.5		398.8
A♯	57	958.1	9.8		948.3	307.8		393.6
B♭	58■	1008.38	7.6		1000.8	298.6		395.9
A𝄪	59	1028.77		1.3	1030.1	293.6		389.1
B	60■	1079.05		1.3	1080.3	285.2	382.3	
c♭	61	1129.33	25.8		1103.5	281.4		397.3
B♯	62	1149.72	1.3		1148.4	274.2	381.3	
c	63■	1200	2.9		1197.1	266.6	385.3	
d𝄫	64	1250.28	30.3		1220.0	263.1		
c♯	65■	1270.67	6.3		1264.6	256.4		
d♭	66	1320.95	11.2		1309.8	249.8		
c𝄪	67	1341.34		0.6	1341.9	245.2		
d	68■	1391.62		4.1	1395.7	237.7		
e𝄫	69	1441.90	22.7		1419.2	234.5		
d♯	70	1462.29		0.2	1462.5	228.7		
e♭	71■	1512.57	11.8		1500.8	223.7		
d𝄪	72	1532.96	3.3		1529.7	220.0		
e	73■	1583.24	1.0		1582.4	213.4	ave.380	

Table 3. Comparison of 12-step sizes, 2/7 comma and 1/4 comma tunings
Normal 12-step sizes in black, abnormal ("quirk") sizes in **red** (BOLD).

	no.	2/7 MT cents	2/7 MT 12-step				1/4 MT 12-step	1/4 MT cents
C	32	0	453.9				462.4	0.00
D𝄫	33	50.28	↓	453.9			462.4	41.06
C♯	34■	70.67		↓	**483.9**		**468.4**	76.05
D♭	35	120.95			↓			117.11
C𝄪	36	141.34				**483.9**		152.10
D	37■	191.62						193.16
E𝄫	38	241.90						234.22
D♯	39	262.29				**483.9**		269.21
E♭	40■	312.57						310.26
D𝄪	41	332.96				**483.9**		345.26
E	42■	383.24						386.31
F♭	43	433.52						427.37
E♯	44	453.91	↓			**483.9**		462.36
F	45■	504.19		↓				503.42
G𝄫	46	554.47			↓			544.48
F♯	47■	574.86						579.47
G♭	48	625.14						620.53
F𝄪	49	645.53				**483.9**		655.52
G	50■	695.81						696.58
A𝄫	51	746.09						737.64
G♯	52■	766.48				**483.9**		772.63
A♭	53	816.76						813.69
G𝄪	54	837.15				**483.9**		848.68
A	55■	887.43						889.74
B𝄫	56	937.71						930.79
A♯	57	958.1				**483.9**		965.78
B♭	58■	1008.38						1006.84
A𝄪	59	1028.77				**483.9**		1041.83
B	60■	1079.05						1082.89
c♭	61	1129.33						1123.95
B♯	62	1149.72						1158.94
c	63■	1200						1200.00
d𝄫	64	1250.28						1241.06
c♯	65■	1270.67						1276.05
d♭	66	1320.95						1317.11
c𝄪	67	1341.34						1352.10
d	68■	1391.62						1393.16
e𝄫	69	1441.90						1434.22
d♯	70	1462.29						1469.21
e♭	71■	1512.57						1510.26
d𝄪	72	1532.96						1545.26
e	73■	1583.24						1586.31

Table 3A. 12-step sizes 2/7 comma and Just Intonation

	no.	FH cents	2/7 MT cents	12 step	Just Intonation	12 step	Mono-chord
C	32	0	0	453.91	0	456.97	457
				-		-	-
C♯	34■	79.6	70.67	-	70.67	-	-
D♭	35	125.6	120.95	453.91	133.24	435.43	455.6
C♯♯	36	154.3	141.34	483.80		489.95	469.8
D	37■	202.3	191.62	453.91	203.91	435.43	450.0
				-		-	-
D♯	39	265.2	262.29	-	274.58	-	-
E♭	40■	318.7	312.57	453.91	315.64	456.99	446.2
D♯♯	41	343.5	332.96	483.80	345.25	468.44	470.1
E	42■	385.7	383.24	453.91	386.30	457.00	451.5
F♭	43	412.7	433.52	453.91	427.33	457.03	-
E♯	44	457.0	453.91	-	456.97	-	-
F	45■	509.5	504.19	453.91	498.00	457.03	438.8
				-		-	-
F♯	47■	581.2	574.86	453.91	568.67	457.03	448.9
G♭	48	624.1	625.14	453.91	631.29	456.98	456.2
F♯♯	49	652.3	645.53	483.80	639.44	489.99	451.2
G	50■	699.4	695.81	453.91	701.96	456.98	449.0
				-		-	-
G♯	52■	764.9	766.48	-	772.63	-	-
A♭	53	813.6	816.76	453.91	813.69	456.98	451.0
G♯♯	54	837.2	837.15	483.80	843.30	489.94	472.6
A	55■	884.3	887.43	453.91	884.36	456.98	457.6
				-		-	-
A♯	57	948.3	958.1	-	955.03	-	-
B♭	58■	1000.8	1008.38	453.91	1017.6	456.98	461.7
A♯♯	59	1030.1	1028.77	483.80	1025.7	489.94	470.7
B	60■	1080.3	1079.05	453.91	1088.27	456.98	449.4
c♭	61	1103.5	1129.33	453.91	1129.33	456.97	-
B♯	62	1148.4	1149.72		1158.94		
c	63■	1197.1	1200		1200.00		

Key:
Column 5. The 12-step size for 2/7 comma tuning
Column 6. Just intonation in cents
Column 7. The 12-step size for just intonation
Column 8. The 12-step sizes of the monochord frets

The C♭, F♭ and ♭♭ notes have been omitted from the monochord in this table because they are obviously defective and are considered separately.

There are differences in some just intonation 12-step sizes (0.01 - 0.06 cents) through rounding up or down, or the limited resolution of the original calculation. These sizes (e.g. 456.97 - 457.03 cents) are all essentially the same.

Table 3B. 25:24 semitone size [♭♭ omitted = (...)]

	no.	2/7 MT cents	♯ asc	♭ desc	- (22)	+ (18)	FH cents	FH 2015 mm	1/4 MT cents
C	32	0	↓	↑			0	532.3	
D♭♭	33	50.28		(...)	31.3		19.0	526.5	41.1
C♯	34■	70.67	79.6			8.9	79.6	508.4	76.1
D♭	35	120.95		76.8		4.7	125.6	495.1	117.1
C♯♯	36	141.34	74.7			13.0	154.3	486.9	152.1
D	37■	191.62				10.7	202.3	473.6	193.2
E♭♭	38	241.90		(...)	12.7		229.2	466.3	234.2
D♯	39	262.29	62.9			2.9	265.2	456.7	269.2
E♭	40■	312.57		67.0		5.7	318.7	442.8	310.3
D♯♯	41	332.96	78.3			10.5	343.5	436.5	345.3
E	42■	383.24				2.5	385.7	426.0	386.3
F♭	43	433.52		96.8	20.8		412.7	419.4	427.4
E♯	44	453.91	71.3			3.1	457.0	408.8	462.4
F	45■	504.19				5.3	509.5	396.6	503.4
G♭♭	46	554.47		(...)	16.9		537.6	390.2	544.5
F♯	47■	574.86	71.7			6.3	581.2	380.5	579.5
G♭	48	625.14		75.3	1.0		624.1	371.2	620.5
F♯♯	49	645.53	71.1			6.8	652.3	365.2	655.5
G	50■	695.81				3.6	699.4	355.4	696.6
A♭♭	51	746.09		(...)	20.2		725.9	350.0	737.6
G♯	52■	766.48	65.5		1.6		764.9	342.2	772.6
A♭	53	816.76		70.7	3.2		813.6	332.7	813.7
G♯♯	54	837.15	72.3			0.1	837.2	328.2	848.7
A	55■	887.43			3.1		884.3	319.4	889.7
B♭♭	56	937.71		(...)	26.7		911.0	314.5	930.8
A♯	57	958.1	64.0		9.8		948.3	307.8	965.8
B♭	58■	1008.38		79.5	7.6		1000.8	298.6	1006.8
A♯♯	59	1028.77	81.8			1.3	1030.1	293.6	1041.8
B	60■	1079.05				1.3	1080.3	285.2	1082.9
c♭	61	1129.33		93.6	25.8		1103.5	281.4	1124.0
B♯	62	1149.72	68.1		1.3		1148.4	274.2	1158.9
c	63■	1200			2.9		1197.1	266.6	1200.0
d♭♭	64	1250.28		(...)	30.3		1220.0	263.1	1241.1
c♯	65■	1270.67	67.5		6.3		1264.6	256.4	1276.1
d♭	66	1320.95		85.9	11.2		1309.8	249.8	1317.1
c♯♯	67	1341.34	77.3			0.6	1341.9	245.2	1352.1
d	68■	1391.62	↓			4.1	1395.7	237.7	1393.2
e♭♭	69	1441.90		(...)	22.7		1419.2	234.5	1434.2
d♯	70	1462.29	66.8			0.2	1462.5	228.7	1469.2
e♭	71■	1512.57		81.6	11.8		1500.8	223.7	1510.3
d♯♯	72	1532.96	67.2		3.3		1529.7	220.0	1545.3
e	73■	1583.24		↑	1.0		1582.4	213.4	1586.3

Table 3C. Theoretically correct fret positions in mm for 2/7 comma (column 5)

C corrected to 533.2 mm

NOTE	key no.	2/7 MT cents	2/7 MT RATIO	2/7 MT mm	mono. mm	- mm low	±0.5 mm	+mm high	mono. cents
C	32	0	0	533.2	532.3	-	-	-	0
D𝄫	33	50.28	1.02947	517.9	526.5	(8.6)			19.0
C♯	34■	70.67	1.04167	511.9	508.4			3.5	79.6
D♭	35	120.95	1.07236	497.2	495.1		-0.1		125.6
C𝄪	36	141.34	1.08507	491.4	486.9			4.5	154.3
D	37■	191.62	1.11704	477.3	473.6			3.7	202.3
E𝄫	38	241.90	1.14996	463.7	466.3	(2.6)			229.2
D♯	39	262.29	1.16358	458.2	456.7			1.5	265.2
E♭	40■	312.57	1.19787	445.1	442.8			2.3	318.7
D𝄪	41	332.96	1.21206	439.9	436.5			3.4	343.5
E	42■	383.24	1.24778	427.3	426.0			1.3	385.7
F♭	43	433.52	1.28455	415.1	419.4	(4.3)			412.7
E♯	44	453.91	1.29977	410.2	408.8			1.4	457.0
F	45■	504.19	1.33807	398.5	396.6			1.9	509.5
G𝄫	46	554.47	1.37751	387.1	390.2	(3.1)			537.6
F♯	47■	574.86	1.39383	382.6	380.5			2.1	581.2
G♭	48	625.14	1.43490	371.6	371.2		0.4+		624.1
F𝄪	49	645.53	1.45190	367.2	365.2			2.0	652.3
G	50■	695.81	1.49469	356.7	355.4			1.3	699.4
A𝄫	51	746.09	1.53873	346.5	350.0	(3.5)			725.9
G♯	52■	766.48	1.55696	342.5	342.2		0.3+		764.9
A♭	53	816.76	1.60284	332.7	332.7		±0		813.6
G𝄪	54	837.15	1.62183	328.8	328.2			0.6	837.2
A	55■	887.43	1.66963	319.4	319.4		±0		884.3
B𝄫	56	937.71	1.71883	310.2	314.5	(4.3)			911.0
A♯	57	958.1	1.73919	306.6	307.8	1.2			948.3
B♭	58■	1008.38	1.79044	297.8	298.6	0.8			1000.8
A𝄪	59	1028.77	1.81166	294.3	293.6			0.7	1030.1
B	60■	1079.05	1.86504	285.9	285.2			0.7	1080.3
c♭	61	1129.33	1.92000	277.7	281.4	(3.7)			1103.5
B♯	62	1149.72	1.94275	274.5	274.2		0.3+		1148.4
c	63■	1200	2.00000	266.6	266.6		±0		1197.1
d𝄫	64	1250.28	2.05894	259.0	263.1	(4.1)			1220.0
c♯	65■	1270.67	2.08333	255.9	256.4		-0.5		1264.6
d♭	66	1320.95	2.14472	248.6	249.8	1.2			1309.8
c𝄪	67	1341.34	2.17013	245.7	245.2		0.5+		1341.9
d	68■	1391.62	2.23408	238.7	237.7			1.0	1395.7
e𝄫	69	1441.90	2.29992	231.8	234.5	(2.7)			1419.2
d♯	70	1462.29	2.32717	229.1	228.7		0.4+		1462.5
e♭	71■	1512.57	2.39575	222.6	223.7	1.1			1500.8
d𝄪	72	1532.96	2.42413	220.0	220.0		±0		1529.7
e	73■	1583.24	2.49557	213.7	213.4		0.3+		1582.4

Table 3D. Theoretically correct fret positions in mm for Just Intonation

C corrected to 533.2 mm

NOTE	key no.	2/7 MT cents	Just Int. RATIO	Just Int mm	mono. mm	- mm low	±0.5 mm	+mm high	mono. cents
C	32	0	0	533.2	532.3	-	-	-	0
D𝄫	33	50.28		514.3	526.5	(12.2)			19.0
C♯	34■	70.67		511.9	508.4			3.5	79.6
D♭	35	120.95		493.7	495.1	1.4			125.6
C𝄪	36	141.34		491.4	486.9			4.5	154.3
D	37■	191.62		474.0	473.6		0.4+		202.3
E𝄫	38	241.90		462.8	466.3	(3.5)			229.2
D♯	39	262.29		455.0	456.7	1.7			265.2
E♭	40■	312.57		444.3	442.8			1.5	318.7
D𝄪	41	332.96		436.8	436.5		0.3+		343.5
E	42■	383.24		426.6	426.0	0.6			385.7
F♭	43	433.52		416.6	419.4	(2.8)			412.7
E♯	44	453.91		409.5	408.8			0.7	457.0
F	45■	504.19		399.9	396.6			3.3	509.5
G𝄫	46	554.47		385.7	390.2	(4.5)			537.6
F♯	47■	574.86		383.9	380.5			3.4	581.2
G♭	48	625.14		370.3	371.2	0.9			624.1
F𝄪	49	645.53		368.5	365.2			3.3	652.3
G	50■	695.81		355.5	355.4		0.1+		699.4
A𝄫	51	746.09		347.1	350.0	(2.9)			725.9
G♯	52■	766.48		341.2	342.2	1.0			764.9
A♭	53	816.76		333.3	332.7			0.6	813.6
G𝄪	54	837.15		327.6	328.2			0.6	837.2
A	55■	887.43		319.9	319.4		0.5+		884.3
B𝄫	56	937.71		308.6	314.5	(5.9)			911.0
A♯	57	958.1		307.1	307.8	0.7			948.3
B♭	58■	1008.38		296.2	298.6	2.4			1000.8
A𝄪	59	1028.77		294.8	293.6			1.2	1030.1
B	60■	1079.05		284.4	285.2	0.8			1080.3
c♭	61	1129.33		277.7	281.4	(3.7)			1103.5
B♯	62	1149.72		273.0	274.2	1.2			1148.4
c	63■	1200		266.6	266.6		±0.0		1197.1
d𝄫	64	1250.28		257.1	263.1	(6.0)			1220.0
c♯	65■	1270.67		255.9	256.4		-0.5		1264.6
d♭	66	1320.95		246.9	249.8	2.9			1309.8
c𝄪	67	1341.34		245.7	245.2		0.5+		1341.9
d	68■	1391.62		237.0	237.7	0.7			1395.7
e𝄫	69	1441.90		231.4	234.5	(3.1)			1419.2
d♯	70	1462.29		227.5	228.7	1.2			1462.5
e♭	71■	1512.57		222.2	223.7	1.5			1500.8
d𝄪	72	1532.96		218.4	220.0	1.6			1529.7
e	73■	1583.24		213.3	213.4		-0.1		1582.4

Table 3E. Theoretically correct fret positions in mm for 1/4 comma meantone

C corrected to 533.2 mm

NOTE	key no.	1/4 MT cents	1/4 MT RATIO	1/4 MT mm	mono. mm	- mm low	±0.5 mm	+mm high	mono. cents
C	32	0	0	533.2	532.3	-	-	-	0
D𝄫	33	41.1	1.02402	520.7	526.5	(5.8)			19.0
C♯	34■	76.1	1.04494	510.3	508.4			1.9	79.6
D♭	35	117.1	1.06998	498.3	495.1			3.2	125.6
C𝄪	36	152.1	1.09183	488.4	486.9			1.5	154.3
D	37■	193.2	1.11806	476.9	473.6			3.3	202.3
E𝄫	38	234.2	1.14486	465.7	466.3	0.6			229.2
D♯	39	269.2	1.16824	456.4	456.7		-0.3		265.2
E♭	40■	310.3	1.19630	445.7	442.8			2.9	318.7
D𝄪	41	345.3	1.22073	436.8	436.5		0.3+		343.5
E	42■	386.3	1.25000	426.6	426.0			0.6	385.7
F♭	43	427.4	1.28002	416.6	419.4	(2.8)			412.7
E♯	44	462.4	1.30616	408.2	408.8	0.6			457.0
F	45■	503.4	1.33746	398.7	396.6			2.1	389.3
G𝄫	46	544.5	1.36950	389.3	390.2	(0.9)			537.6
F♯	47■	579.5	1.39757	381.5	380.5			1.0	581.2
G♭	48	620.5	1.43106	372.6	371.2			1.4	624.1
F𝄪	49	655.5	1.46029	365.1	365.2		-0.1		652.3
G	50■	696.6	1.49537	356.6	355.4			1.2	699.4
A𝄫	51	737.6	1.53120	348.2	350.0	1.8			725.9
G♯	52■	772.6	1.56248	341.3	342.2	0.9			764.9
A♭	53	813.7	1.60001	333.2	332.7		0.5+		813.6
G𝄪	54	848.7	1.63269	326.6	328.2	1.6			837.2
A	55■	889.7	1.67182	318.9	319.4		-0.5		884.3
B𝄫	56	930.8	1.71198	311.5	314.5	(3.0)			911.0
A♯	57	965.8	1.74694	305.2	307.8	2.6			948.3
B♭	58■	1006.8	1.78881	298.1	298.6		-0.5		1000.8
A𝄪	59	1041.8	1.82534	292.1	293.6	1.5			1030.1
B	60■	1082.9	1.86919	285.3	285.2		0.1+		1080.3
c♭	61	1124.0	1.91410	278.6	281.4	(2.8)			1103.5
B♯	62	1158.9	1.95308	273.0	274.2	1.2			1148.4
c	63■	1200.0	2.00000	266.6	266.6		±0		1197.1
d𝄫	64	1241.1	2.04805	260.3	263.1	(2.8)			1220.0
c♯	65■	1276.1	2.08988	255.1	256.4	1.3			1264.6
d♭	66	1317.1	2.13996	249.2	249.8	0.6			1309.8
c𝄪	67	1352.1	2.18366	244.2	245.2	1.0			1341.9
d	68■	1393.2	2.23612	238.4	237.7			0.7	1395.7
e𝄫	69	1434.2	2.28971	232.9	234.5	(1.6)			1419.2
d♯	70	1469.2	2.33647	228.2	228.7		-0.5		1462.5
e♭	71■	1510.3	2.39261	222.9	223.7	0.8			1500.8
d𝄪	72	1545.3	2.44147	218.4	220.0	1.6			1529.7
e	73■	1586.3	2.50000	213.3	213.4		-0.1		1582.4

Table 3F. Comparison of 2/7 comma, just intonation, and 1/4 comma meantone

C corrected to 533.2 mm

			2/7 comma			just intonation			1/4 comma		
NOTE	key no.	mono. mm	- mm low	±0.5 mm	+mm high	- mm low	±0.5 mm	+mm high	- mm low	±0.5 mm	+mm high
C	32	533.2	-	-	-	-	-	-	-	-	-
D𝄫	33	526.5	(8.6)			(12.2)			(5.8)		
C♯	34■	508.4			3.5			3.5			1.9
D♭	35	495.1		-0.1		1.4					3.2
C𝄪	36	486.9			4.5			4.5			1.5
D	37■	473.6			3.7		0.4+				3.3
E𝄫	38	466.3	(2.6)			(3.5)			0.6		
D♯	39	456.7			1.5	1.7				-0.3	
E♭	40■	442.8			2.3			1.5			2.9
D𝄪	41	436.5			3.4		0.3+			0.3+	
E	42■	426.0			1.3	0.6					0.6
F♭	43	419.4	(4.3)			(2.8)			(2.8)		
E♯	44	408.8			1.4			0.7	0.6		
F	45■	396.6			1.9			3.3			2.1
G𝄫	46	390.2	(3.1)			(4.5)			(0.9)		
F♯	47■	380.5			2.1			3.4			1.0
G♭	48	371.2		0.4+		0.9					1.4
F𝄪	49	365.2			2.0			3.3		-0.1	
G	50■	355.4			1.3		0.1+				1.2
A𝄫	51	350.0	(3.5)			(2.9)			1.8		
G♯	52■	342.2		0.3+		1.0			0.9		
A♭	53	332.7		±0				0.6		0.5+	
G𝄪	54	328.2			0.6			0.6	1.6		
A	55■	319.4		±0			0.5+			-0.5	
B𝄫	56	314.5	(4.3)			(5.9)			(3.0)		
A♯	57	307.8	1.2			0.7			2.6		
B♭	58■	298.6	0.8			2.4				-0.5	
A𝄪	59	293.6			0.7			1.2	1.5		
B	60■	285.2			0.7	0.8				0.1+	
c♭	61	281.4	(3.7)			(3.7)			(2.8)		
B♯	62	274.2		0.3+		1.2			1.2		
c	63■	266.6		±0			±0			±0	
d𝄫	64	263.1	(4.1)			(6.0)			(2.8)		
c♯	65■	256.4		-0.5			-0.5		1.3		
d♭	66	249.8	1.2			2.9			0.6		
c𝄪	67	245.2		0.5+			0.5+		1.0		
d	68■	237.7			1.0	0.7					0.7
e𝄫	69	234.5	(2.7)			(3.1)			(1.6)		
d♯	70	228.7		0.4+		1.2				-0.5	
e♭	71■	223.7	1.1			1.5			0.8		
d𝄪	72	220.0		±0		1.6			1.6		
e	73■	213.4		0.3+			-0.1			-0.1	

Table 3G. Comparison of 2/7 comma, just intonation, and 1/4 comma meantone
simplified to 19-note compass, accidentals only

			2/7 comma			just intonation			1/4 comma		
NOTE	key no.	mono. mm	- mm low	±0.5 mm	+mm high	- mm low	±0.5 mm	+mm high	- mm low	±0.5 mm	+mm high
C	32	533.2	-	-	-	-	-	-	-	-	-
D𝄫	33	526.5									
C♯	34■	508.4			3.5			3.5			1.9
D♭	35	495.1		-0.1		1.4					3.2
C𝄪	36	486.9									
D	37■	473.6									
E𝄫	38	466.3									
D♯	39	456.7			1.5	1.7				-0.3	
E♭	40■	442.8			2.3			1.5			2.9
D𝄪	41	436.5									
E	42■	426.0									
F♭	43	419.4									
E♯	44	408.8			1.4			0.7	0.6		
F	45■	396.6									
G𝄫	46	390.2									
F♯	47■	380.5			2.1			3.4			1.0
G♭	48	371.2		0.4+		0.9					1.4
F𝄪	49	365.2									
G	50■	355.4									
A𝄫	51	350.0									
G♯	52■	342.2		0.3+		1.0			0.9		
A♭	53	332.7		±0				0.6		0.5+	
G𝄪	54	328.2									
A	55■	319.4									
B𝄫	56	314.5									
A♯	57	307.8	1.2			0.7			2.6		
B♭	58■	298.6	0.8			2.3				-0.5	
A𝄪	59	293.6									
B	60■	285.2									
c♭	61	281.4									
B♯	62	274.2		0.3+		1.2			1.2		
c	63■	266.6									
d𝄫	64	263.1									
c♯	65■	256.4		-0.5			-0.5		1.3		
d♭	66	249.8	1.2			2.9			0.6		
c𝄪	67	245.2									
d	68■	237.7									
e𝄫	69	234.5									
d♯	70	228.7		0.4+		1.2				-0.5	
e♭	71■	223.7	1.1			1.5			0.8		
d𝄪	72	220.0									
e	73■	213.4									

Table 3H. Comparison of 2/7 comma, just intonation, and 1/4 comma meantone
simplified to diatonic notes only

			2/7 comma			just intonation			1/4 comma		
NOTE	key no.	mono. mm	- mm low	±0.5 mm	+mm high	- mm low	±0.5 mm	+mm high	- mm low	±0.5 mm	+mm high
C	32	533.2	-	-	-	-	-	-	-	-	-
D𝄫	33	526.5									
C♯	34■	508.4									
D♭	35	495.1									
C𝄪	36	486.9									
D	37■	473.6			3.7		0.4+				3.3
E𝄫	38	466.3									
D♯	39	456.7									
E♭	40■	442.8									
D𝄪	41	436.5									
E	42■	426.0			1.3	0.6					0.6
F♭	43	419.4									
E♯	44	408.8									
F	45■	396.6			1.9			3.3			2.1
G𝄫	46	390.2									
F♯	47■	380.5									
G♭	48	371.2									
F𝄪	49	365.2									
G	50■	355.4			1.3		0.1+				1.2
A𝄫	51	350.0									
G♯	52■	342.2									
A♭	53	332.7									
G𝄪	54	328.2									
A	55■	319.4		±0			0.5+			-0.5	
B𝄫	56	314.5									
A♯	57	307.8									
B♭	58■	298.6									
A𝄪	59	293.6									
B	60■	285.2			0.7	0.8				0.1+	
c♭	61	281.4									
B♯	62	274.2									
c	63■	266.6		±0			±0			±0	
d𝄫	64	263.1									
c♯	65■	256.4									
d♭	66	249.8									
c𝄪	67	245.2									
d	68■	237.7			1.0	0.7					0.7
e𝄫	69	234.5									
d♯	70	228.7									
e♭	71■	223.7									
d𝄪	72	220.0									
e	73■	213.4		0.3+			-0.1			-0.1	

Table 3l. Monochord compared with 2/7 comma and Just Intonation

(19-note range, no ♭♭ notes)

	no.	FH cents	2/7 MT cents	- (22)	+ (18)	8 14	Just cents	-	+	8 8
C	32	0	0				0			
C♯	34■	79.6	70.67		8.9		70.67		8.9	
D♭	35	125.6	120.95		4.7	+	133.24	7.6		
C♯♯	36	154.3	141.34							
D	37■	202.3	191.62		10.7		203.91	1.6		+
D♯	39	265.2	262.29		2.9	+	274.58	9.4		
E♭	40■	318.7	312.57		5.7		315.64		3.1	+
D♯♯	41	343.5	332.96							
E	42■	385.7	383.24		2.5		386.30	0.6		+
F♭	43	412.7	433.52				427.33			
E♯	44	457.0	453.91		3.1		456.97	0	0	+
F	45■	509.5	504.19		5.3	+	498.00		11.5	
F♯	47■	581.2	574.86		6.3	+	568.67		12.5	
G♭	48	624.1	625.14	1.0			631.29	7.2		
F♯♯	49	652.3	645.53							
G	50■	699.4	695.81		3.6		701.96	2.6		+
G♯	52■	764.9	766.48	1.6		+	772.63	7.7		
A♭	53	813.6	816.76	3.2			813.69	0.1		+
G♯♯	54	837.2	837.15							
A	55■	884.3	887.43	3.1			884.36	0.1		+
A♯	57	948.3	958.1	9.8			955.03	6.7		+
B♭	58■	1000.8	1008.38	7.6		+	1017.6	16.8		
A♯♯	59	1030.1	1028.77							
B	60■	1080.3	1079.05		1.3	+	1088.27	8.0		
c♭	61	1103.5	1129.33				1129.33			
B♯	62	1148.4	1149.72	1.3		+	1158.94	10.5		
c	63■	1197.1	1200	2.9			1200.00	2.9		
c♯	65■	1264.6	1270.67	6.3			1270.67	6.3		
d♭	66	1309.8	1320.95	11.2		+	1333.24	23.4		
c♯♯	67	1341.9	1341.34							
d	68■	1395.7	1391.62		4.1	+	1403.91	8.2		
d♯	70	1462.5	1462.29		0.2	+	1474.58	12.1		
e♭	71■	1500.8	1512.57	11.8		+	1515.64	14.8		
d♯♯	72	1529.7	1532.96							
e	73■	1582.4	1583.24	0.8		+	1586.30	3.9		

Table 4. Zarlino's 2/7 comma meantone division
Le Istitutioni Harmoniche, 1558, p. 130 (page not numbered in the original edition)
[by permission of the Bayerische Staatsbibliothek]
See Table 4A for an explanation of the scheme.

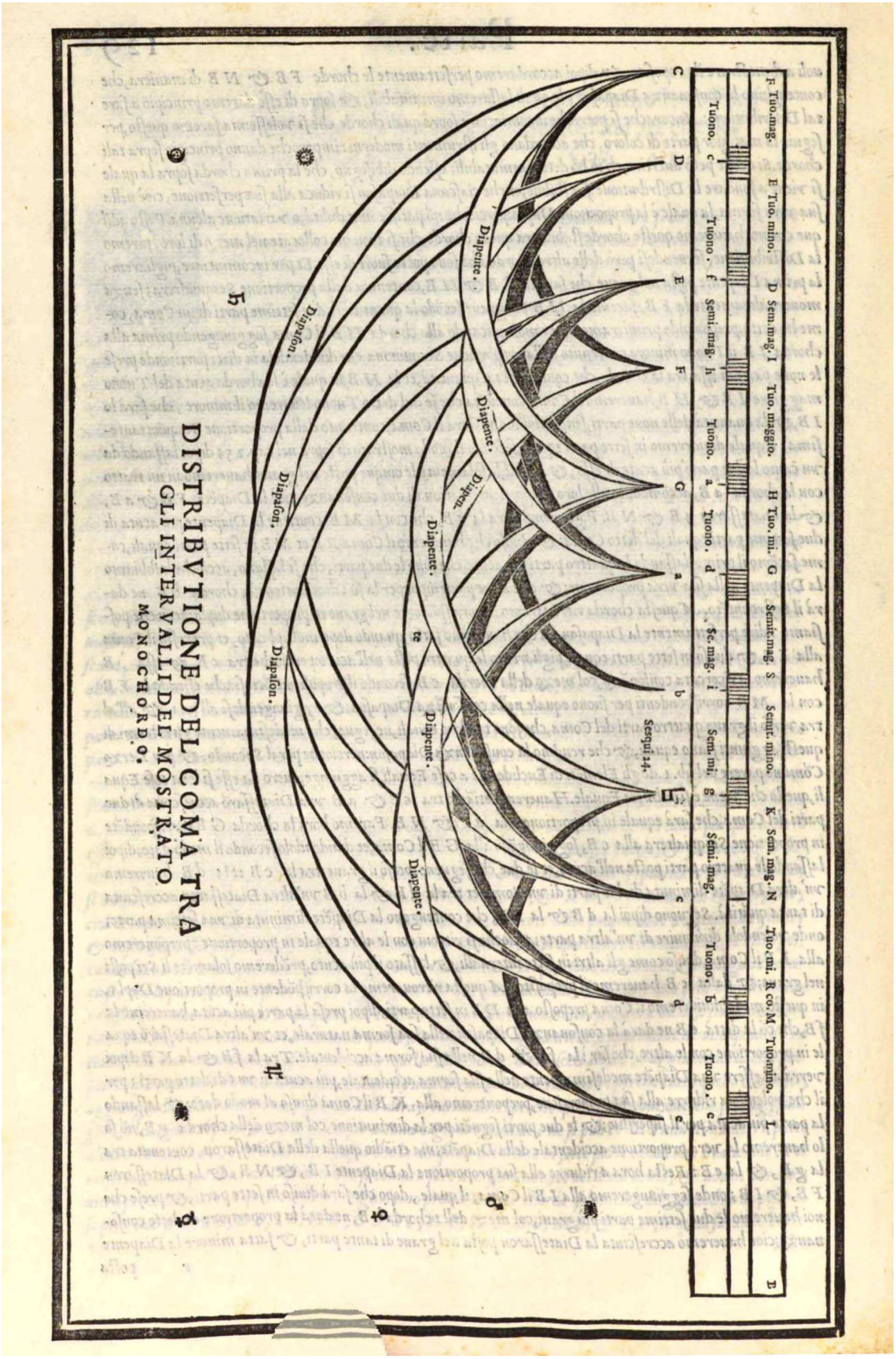


Table 4A. Zarlino's instructions for the 2/7 comma tuning from his diagram, p. 130

The rectangular box (within the outer frame) containing the information in Zarlino's diagram measures 20mm x 261mm in a printout from a PDF copy of the 1573 edition, but the absolute size of the original is not known to me. The discussion here is of the 1558 edtion.

The top row of letters (in the box diagram) demarcates the size of the just intonation interval, which is to be compensated with some part of the comma. The letters are placed exactly over the start or end of an interval, drawn in the second row.

The second row consists of two parallel lines and contains the *schematic* position of a comma, divided into 7 parts with 8 vertical lines, relative to the just intonation intervals labelled in the top row.

The third row, with letters, designates the exact position of the beginning or end of a compensated interval. The exact part of the comma required, as an addition or subtraction from the just intonation interval, is shown by a small line below the second row. All these letters combine, without gaps or errors, to form the octave correctly divided into the respective intervals in Zarlino's 2/7 comma tuning.

The fourth row, with letters, under the box diagram gives the monohord note name, placed below the letters of the third row.

The editions of 1558, 1561 and 1562 were all printed from the same block. In 1573 rows 2-3 remained unchanged, but the title was modified: COMA became COMMA and "DIATONICO SINTONO" was added to the description, but the astrological symbols for the planets were removed. More significantly, type was re-cast in the top row with the effect that from G to M all letters were too far to the right, and therefore no longer correctly corresponded to the just intonation intervals. The binding of the diagram in the book was reversed. In 1589 an attempt was apparently made to correct the top row of letters, moving them leftwards, but not by enough to restore the correct version of 1558.

monochord interval	interval origin	required change	top row size label	comma modification	2/7 tuning designation
C-D	TM	-4/7	FE	-4/7 @ E	[F]c
D-E	Tm	+3/7	ED	+4/7 @ E -1/7 @ D	cf
E-F	SM	+3/7	DI	+1/7 @ D +2/7 @ I	fh
F-G	TM	-4/7	IH	-2/7 @ I -2/7 @ H	ha
G-a	Tm	+3/7	HG	+2/7 @ H +1/7 @ G	ad
a-b (b rotondo)	SM	+3/7	GS	-1/7 @ G +4/7 @ S	di
b-♮ (b quadro)	Sm	none	SK	none	ig
♮-c	SM	+3/7	KN	+3/7 @ K	g[N]
c-d	Tm	+3/7	NR	+3/7 @ R	[N]b
d-e	Tm	+3/7	ML	+4/7 @ M -1/7 @ L	bc

The following abbreviations are used in the table above

TM = Tuono maggiore	9:8	[Tuo. mag. Tuo. maggio.]
Tm = Tuono minore	10:9	[Tu. mi. Tuo. mi. Tuo. mino.]
SM = Semituono maggiore	16:15	[Sem. mag. Semi. mag. Semit. mag]
Sm = Semituono minore	25:24	[Semit. mino. Sem. mi.]

The abbreviations in square brackets above are the names used in the diagram.

The first difficulty in comprehension is that just intonation intervals labelled in the first row and drawn in the second row are not shown in proportion to their size: tuono minore is as large as tuono maggiore, and the two semitones are also equally sized.

The second difficulty concerns the comma, divided into 7 parts (with 8 lines). It appears as if there is a comma to be found *between* each interval. Of course, this cannot be, except at C-E, where the syntonic comma is between the tuono maggiore and tuono minore. It could be placed between F-A, but Zarlino uses a different construction there. What the reader should understand is that Zarlino *shows* the whole comma, but takes only the part of it required at that position; the unused part is redundant

The third difficulty is that in the resulting intervals in the fourth row, the compensated tones C-D and D-E, which are now of equal size, are shown as unequal. In fact what was the tuono minore has become larger than the original tuono maggiore!

Despite these inelegancies of displaying the system, all the correct parts of the comma have been added or subtracted as necessary. The labels for the just intonation intervals in the top row are all correctly identified in the 1558-1562 editions, but from 1573 onward some are incorrectly placed.

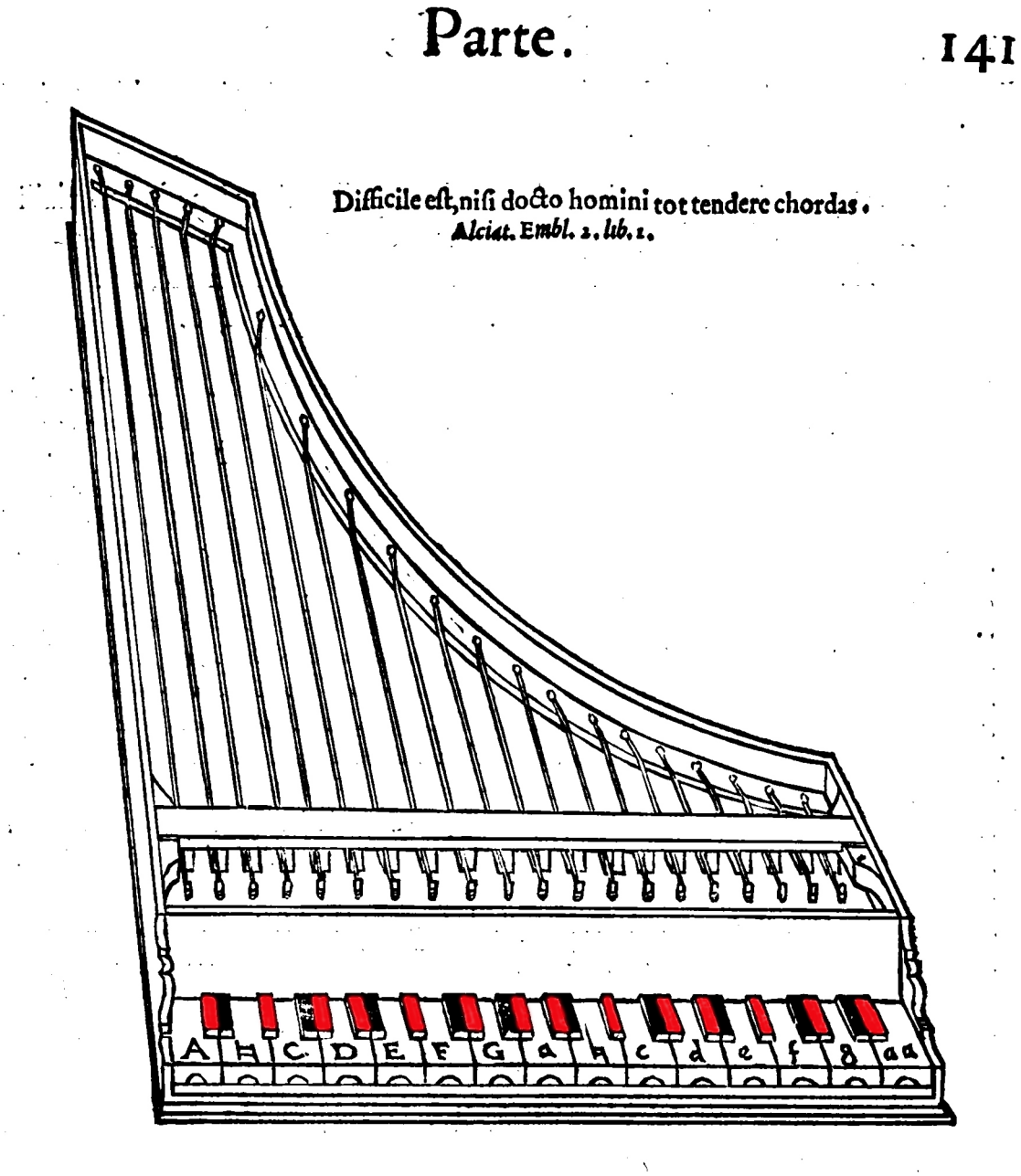
The construction of C-D, by subtracting 4/7 comma from a tuono maggiore, uses a different method than at c, an octave higher, which starts from two tuono minore and adds 3/7 comma, but both methods are correct. As described in note 26, for the construction above c, the intervals major third (ratio 5:4) and Pythagorean minor third (ratio 32:27) from B construct the distances RB and MB respectively.

Table 4B. Zarlino, 1589, Sopplimenti musicali, p. 155

Systema massimo arteficiale del Naturale ò Syntono diatonico; accresciuto con molte chorde, per l'acquisto di molte Consonanze		
33. aa.	*4320*	*PARTE ACVTA*
32.	*# 4608*	*Semituono. 16.15*
31. g.	*4800*	*Semituono. 25.24*
30.	*# 5120*	*Semituono. 16.15*
29.	*# 5184*	*Comma. 81.80*
28. f.	*5400*	*Semituono. 25.24*
27. e.	*5760*	*Semituono. 16.15*
26.	*b. 6000*	*Semituono. 25.24*
25.	*b. 6075*	*Comma. 81.80*
24. d.	*6400*	*Semituono. 256.243*
23.	*d 6480*	*Comma. 81.80*
22.	*# 6912*	*Semituono. 16.15*
21. c.	*7200*	*Semituono. 25.24*
20. ♮	*7680*	*Semituono. 16.15*
19.	*b. 8000*	*Semituono. 25.24*
18. b.	*8100*	*Comma. 81.80*
17. a.	*8640*	*Semituono. 16.15*
16.	*# 9216*	*Semituono. 16.15*
15. G.	*9600*	*Semituono. 25.24*
14.	*# 10240*	*Semituono. 16.15*
13.	*# 10368*	*Comma. 81.80*
12. F.	*10800*	*Semituono. 25.24*
11. E.	*11520*	*Semituono. 16.15*
10.	*b. 12000*	*Semituono. 25.24*
9.	*b. 12150*	*Comma. 81.80*
8. D.	*12800*	*Semituono. 256.243*
7.	*D 12960*	*Comma. 81.80*
6.	*# 13834 [recte:13824]*	*Semituono. 16.15*
5. C.	*14400*	*Semituono. 25.24*
4. B.	*15360*	*Semituono. 16.15*
3.	*b. 16000*	*Semituono. 25.24*
2.	*b. 16200*	*Comma. 81.80*
1. A.	*17280*	*Semituono. 16.15*
		PARTE GRAVE

The contents of the original plate has been typed into a more easily readable format. The Greek names of the intervals have been omitted from column 2.

Table 4C. Zarlino's enharmonic harpsichord



Part of p. 141 of the first edition of *Le Istitutioni harmoniche* (1558) with the enharmonic notes here coloured red, as Zarlino had intended for his printing.

Uncoloured source:
Google scanned copy provided by the Biblioteca Nazionale Centrale di Roma.

Table 5. Deviations of 1606 monochord fret sizes from 2/7 comma as ± cents
with ranges of the enharmonic harpsichords of 1548, 1601 and 1606

Diatonico		Chromatico		Enharmonico		1548 Fig. 3	1601 Fig. 10	1606 Fig. 11	1606 **monochord**
		ordine B molle							
				[F♭♭]					
				[C♭♭]					
				G♭♭	-16.9			← 1	
				D♭♭	-31.3				
				A♭♭	-20.2				← 1
				E♭♭	-12.7				
				B♭♭	-26.7				
				F♭	-20.8		← 1		
				C♭	-25.8				
				G♭	- 1.0	← 1			
				D♭	+ 4.7				
				A♭	- 3.2				
		E♭	+ 5.7						
		B♭	- 7.6						
F	+ 5.3								
C	start								
G	+ 3.6								
D	+10.7								
A	– 3.1								
E	+ 2.5								
B	+ 1.3								
		F♯	+ 6.3						
		C♯	+ 8.9						
		G♯	- 1.6						
				D♯	+ 2.9				
				A♯	- 9.8				
				E♯	+ 3.1				
				B♯	- 1.3				
				F♯♯	+ 6.8				
				C♯♯	+13.0				
				G♯♯	+ 0.1				
				D♯♯	+10.5				
				A♯♯	+ 1.3	← 24		← 31	
				E♯♯	+ 13.0				
				B♯♯	- 1.4		← 28		← 31
		ordine B quadro							
Diatonico		Chromatico		Enharmonico*					

* Enharmonico according to Zarlino's definition (Zarlino, 1558, p. 109; this article p. 21)

Table 6/1. Zarlino: enharmonic tetrachords, presentation by Ordine

1548 harpsichord range G♭ - A♯♯
realisation of tetrachords with 2/7 comma tuning

24 tetrachords are possible without mistuning
(tetrachords not possible for lack of the correct notes are greyed out in all tables)

KEY
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	F♭♭	F♭		F♭	F	G♭♭	B♭♭
G♭	B♭	C♭♭	C♭		C♭	C	D♭♭	F♭
D♭	F	G♭♭	G♭		G♭	G	A♭♭	C♭
A♭	C	D♭♭	D♭		D♭	D	E♭♭	G♭
E♭	G	A♭♭	A♭		A♭	A	B♭♭	D♭
B♭	D	E♭♭	E♭		E♭	E	F♭	A♭
F	A	B♭♭	B♭		B♭	B	C♭	E♭
C	E	F♭	F		F	F♯	G♭	B♭
G	B	C♭	C		C	C♯	D♭	F
D	F♯	G♭	G		G	G♯	A♭	C
A	C♯	D♭	D		D	D♯	E♭	G
E	G♯	A♭	A		A	A♯	B♭	D
B	D♯	E♭	E		E	E♯	F	A
F♯	A♯	B♭	B		B	B♯	C	E
C♯	E♯	F	F♯		F♯	F♯♯	G	B
G♯	B♯	C	C♯		C♯	C♯♯	D	F♯
D♯	F♯♯	G	G♯		G♯	G♯♯	A	C♯
A♯	C♯♯	D	D♯		D♯	D♯♯	E	G♯
E♯	G♯♯	A	A♯		A♯	A♯♯	B	D♯
B♯	D♯♯	E	E♯		E♯	E♯♯	F♯	A♯
F♯♯	A♯♯	B	B♯		B♯	B♯♯	C♯	E♯

Table 6/1A: required notes for correct tuning with 2/7 comma
A 35-note range is required for correct tuning: F♭♭ - B♯♯

← descending					ascending →			
383.2		50.3	70.7		70.7	50.3	383.2	
C♭	E♭	F♭♭	F♭		F♭	F	G♭♭	B♭♭
G♭	B♭	C♭♭	C♭		C♭	C	D♭♭	F♭

Table 6/2. Zarlino: enharmonic tetrachords, presentation by Ordine

1601 harpsichord range F♭ - B♯♯
mistuning and realisation of tetrachords with 2/7 comma tuning

30 tetrachords are correct, 4 are mistuned, 8 are not possible

KEY
+ oversize intervals (cents)
- undersize intervals (cents)
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	-30 D♯♯	+30 F♭		F♭	F -30	E♯♯ +30	B♭♭
G♭	B♭	-30 A♯♯	+30 C♭		C♭	C –30	B♯♯ +30	F♭
D♭	F	G♭♭	G♭		G♭	G	A♭♭	C♭
A♭	C	D♭♭	D♭		D♭	D	E♭♭	G♭
E♭	G	A♭♭	A♭		A♭	A	B♭♭	D♭
B♭	D	E♭♭	E♭		E♭	E	F♭	A♭
F	A	B♭♭	B♭		B♭	B	C♭	E♭
C	E	F♭	F		F	F♯	G♭	B♭
G	B	C♭	C		C	C♯	D♭	F
D	F♯	G♭	G		G	G♯	A♭	C
A	C♯	D♭	D		D	D♯	E♭	G
E	G♯	A♭	A		A	A♯	B♭	D
B	D♯	E♭	E		E	E♯	F	A
F♯	A♯	B♭	B		B	B♯	C	E
C♯	E♯	F	F♯		F♯	F♯♯	G	B
G♯	B♯	C	C♯		C♯	C♯♯	D	F♯
D♯	F♯♯	G	G♯		G♯	G♯♯	A	C♯
A♯	C♯♯	D	D♯		D♯	D♯♯	E	G♯
E♯	G♯♯	A	A♯		A♯	A♯♯	B	D♯
B♯	D♯♯	E	E♯		E♯	E♯♯	F♯	A♯
F♯♯	A♯♯	B	B♯		B♯	B♯♯	C♯	E♯

Table 6/2A: required notes for correct tuning with 2/7 comma
A 35-note range is required for correct tuning: F♭♭ - B♯♯

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	F♭♭	F♭		F♭	F	G♭♭	B♭♭
G♭	B♭	C♭♭	C♭		C♭	C	D♭♭	F♭

Table 6/3. Zarlino: enharmonic tetrachords, presentation by Ordine

1606 harpsichord range G♭♭ - A♯♯
realisation of tetrachords with 2/7 comma tuning

38 tetrachords are possible without mistuning

KEY
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	F♭♭	F♭		F♭	F	G♭♭	B♭♭
G♭	B♭	C♭♭	C♭		C♭	C	D♭♭	F♭
D♭	F	G♭♭	G♭		G♭	G	A♭♭	C♭
A♭	C	D♭♭	D♭		D♭	D	E♭♭	G♭
E♭	G	A♭♭	A♭		A♭	A	B♭♭	D♭
B♭	D	E♭♭	E♭		E♭	E	F♭	A♭
F	A	B♭♭	B♭		B♭	B	C♭	E♭
C	E	F♭	F		F	F♯	G♭	B♭
G	B	C♭	C		C	C♯	D♭	F
D	F♯	G♭	G		G	G♯	A♭	C
A	C♯	D♭	D		D	D♯	E♭	G
E	G♯	A♭	A		A	A♯	B♭	D
B	D♯	E♭	E		E	E♯	F	A
F♯	A♯	B♭	B		B	B♯	C	E
C♯	E♯	F	F♯		F♯	F♯♯	G	B
G♯	B♯	C	C♯		C♯	C♯♯	D	F♯
D♯	F♯♯	G	G♯		G♯	G♯♯	A	C♯
A♯	C♯♯	D	D♯		D♯	D♯♯	E	G♯
E♯	G♯♯	A	A♯		A♯	A♯♯	B	D♯
B♯	D♯♯	E	E♯		E♯	E♯♯	F♯	A♯
F♯♯	A♯♯	B	B♯		B♯	B♯♯	C♯	E♯

Table 6/3A: required notes for correct tuning with 2/7 comma
A 35-note range is required for correct tuning: F♭♭ - B♯♯

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	F♭♭	F♭		F♭	F	G♭♭	B♭♭
G♭	B♭	C♭♭	C♭		C♭	C	D♭♭	F♭

Table 6/4. Zarlino: enharmonic tetrachords, presentation by Ordine

1606 harpsichord range A𝄫 - B𝄪
realisation of tetrachords with 2/7 comma tuning

38 tetrachords are possible without mistuning, as in Table 6/3, but at different notes

KEY
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	F𝄫	F♭		F♭	F	E𝄪	B𝄫
G♭	B♭	C𝄫	C♭		C♭	C	B𝄪	F♭
D♭	F	G𝄫	G♭		G♭	G	A𝄫	C♭
A♭	C	D𝄫	D♭		D♭	D	E𝄫	G♭
E♭	G	A𝄫	A♭		A♭	A	B𝄫	D♭
B♭	D	E𝄫	E♭		E♭	E	F♭	A♭
F	A	B𝄫	B♭		B♭	B	C♭	E♭
C	E	F♭	F		F	F♯	G♭	B♭
G	B	C♭	C		C	C♯	D♭	F
D	F♯	G♭	G		G	G♯	A♭	C
A	C♯	D♭	D		D	D♯	E♭	G
E	G♯	A♭	A		A	A♯	B♭	D
B	D♯	E♭	E		E	E♯	F	A
F♯	A♯	B♭	B		B	B♯	C	E
C♯	E♯	F	F♯		F♯	F𝄪	G	B
G♯	B♯	C	C♯		C♯	C𝄪	D	F♯
D♯	F𝄪	G	G♯		G♯	G𝄪	A	C♯
A♯	C𝄪	D	D♯		D♯	D𝄪	E	G♯
E♯	G𝄪	A	A♯		A♯	A𝄪	B	D♯
B♯	D𝄪	E	E♯		E♯	E𝄪	F♯	A♯
F𝄪	A𝄪	B	B♯		B♯	B𝄪	C♯	E♯

Table 6/4A: required notes for correct tuning with 2/7 comma
A 35-note range is required for correct 2/7 comma tuning: F𝄫 - B𝄪

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	E♭	F𝄫	F♭		F♭	F	G𝄫	B𝄫
G♭	B♭	C𝄫	C♭		C♭	C	D𝄫	F♭

Table 6/5. Zarlino: enharmonic tetrachords, presentation by Ordine

1548 and 1601 harpsichords
interval sizes resulting from 2/7 comma meantone and actual 1606 monochord tuning

(where cent values are not shown the intervals are unchanged)

KEY
oversize intervals (cents)
undersize intervals (cents)
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		mD 50.3	MD 70.7		MD 70.7	mD 50.3	M III 383.2	
C♭	415.2 E♭	D♯♯ 26.7	69.2 F♭		F♭ 96.8	F 28.1	E♯♯ 410.7	A♯
G♭	376.7 B♭	A♯♯ 29.3	73.4 C♭		C♭ 96.5	C 19	B♯♯ 392.3	F♭
D♭	383.9 F	G♭♭ 28.1	86.5 G♭		G♭	G 26.5	A♭♭ 377.6	C♭
A♭	383.5 C	D♭♭ 19	106.6 D♭		D♭	D 25.7	E♭♭ 396	G♭
E♭	380.7 G	A♭♭ 26.5	87.7 A♭		A♭	A 26.7	B♭♭ 398.8	D♭
B♭	394.9 D	E♭♭ 25.7	90.6 E♭		E♭	E 27	F♭ 400.9	A♭
F	374.8 A	B♭♭ 26.7	89.8 B♭		B♭	B 23.2	C♭ 387.3	E♭
C	385.7 E	F♭ 27	96.8 F		F	F♯	G♭	B♭
G	380.9 B	C♭ 23.2	93.6 C		C	C♯	D♭	F
D	F♯	G♭	G		G	G♯	A♭	C
A	C♯	D♭	D		D	D♯	E♭	G
E	G♯	A♭	A		A	A♯	B♭	D
B	D♯	E♭	E		E	E♯	F	A
F♯	A♯	B♭	B		B	B♯	C	E
C♯	E♯	F	F♯		F♯	F♯♯	G	B
G♯	B♯	C	C♯		C♯	C♯♯	D	F♯
D♯	F♯♯	G	G♯		G♯	G♯♯	A	C♯
A♯	C♯♯	D	D♯		D♯	D♯♯	E	G♯
E♯	G♯♯	A	A♯		A♯	A♯♯	B	D♯
B♯	D♯♯	E	E♯		E♯	E♯♯	F♯	A♯
F♯♯	A♯♯	B	B♯		B♯	B♯♯	C♯	E♯

Table 7/1. Ptolemy-Vicentino: enharmonic tetrachords, presentation by Ordine

1606 Clavemusicum Omnitonum range G𝄫 - A𝄪
mistuning of intervals in 2/7 comma meantone tuning
28 tetrachords are correct, 14 are mistuned

KEY
+ oversize intervals (cents)
- undersize intervals (cents)
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		MD 70.7	mD 50.3		mD 50.3	MD 70.7	M III 383.2	
C♭	E♭	E	F♭		F♭ -30	E♯ +30	G𝄫	B𝄫
G♭	B♭	B	C♭		C♭ -30	B♯ +30	D𝄫	F♭
D♭	F	F♯	G♭		G♭ -30	F𝄪 +30	A𝄫	C♭
A♭	C	C♯	D♭		D♭ -30	C𝄪 +30	E𝄫	G♭
E♭	G	G♯	A♭		A♭ -30	G𝄪 +30	B𝄫	D♭
B♭	D	D♯	E♭		E♭ -30	D𝄪 +30	F♭	A♭
F	A	A♯	B♭		B♭ -30	A𝄪 +30	C♭	E♭
C	E	E♯	F		F	G𝄫	G♭	B♭
G	B	B♯	C		C	D𝄫	D♭	F
D	F♯	F𝄪	G		G	A𝄫	A♭	C
A	C♯	C𝄪	D		D	E𝄫	E♭	G
E	G♯	G𝄪	A		A	B𝄫	B♭	D
B	D♯	D𝄪	E		E	F♭	F	A
F♯	A♯	A𝄪	B		B	C♭	C	E
C♯	E♯	+30 G𝄫	-30 F♯		F♯	G♭	G	B
G♯	B♯	+30 D𝄫	-30 C♯		C♯	D♭	D	F♯
D♯	F𝄪	+30 A𝄫	-30 G♯		G♯	A♭	A	C♯
A♯	C𝄪	+30 E𝄫	-30 D♯		D♯	E♭	E	G♯
E♯	G𝄪	+30 B𝄫	-30 A♯		A♯	B♭	B	D♯
B♯	D𝄪	+30 F♭	-30 E♯		E♯	F	F♯	A♯
F𝄪	A𝄪	+30 C♭	-30 B♯		B♯	C	C♯	E♯

Table 7/1A : A 45-note range is required for correct 2/7 comma tuning : G𝄫♭ - A𝄪♯

← descending					ascending →			
M III 383.2		MD 70.7	mD 50.3		mD 50.3	MD 70.7	M III 383.2	
C♯	E♯	E𝄪	F♯		F♭	G𝄫♭	G𝄫	B𝄫
G♯	B♯	B𝄪	C♯		C♭	D𝄫♭	D𝄫	F♭
D♯	F𝄪	F𝄪♯	G♯		G♭	A𝄫♭	A𝄫	C♭
A♯	C𝄪	C𝄪♯	D♯		D♭	E𝄫♭	E𝄫	G♭
E♯	G𝄪	G𝄪♯	A♯		A♭	B𝄫♭	B𝄫	D♭
B♯	D𝄪	D𝄪♯	E♯		E♭	F𝄫	F♭	A♭
F𝄪	A𝄪	A𝄪♯	B♯		B♭	C𝄫	C♭	E♭

Table 7/2. Ptolemy-Vicentino: enharmonic tetrachords, presentation by Ordine

1606 Clavemusicum Omnitonum range G♭♭ - A♯♯
interval sizes resulting from 2/7 comma meantone and actual 1606 monochord tuning

KEY
oversize intervals (cents)
undersize intervals (cents)
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		MD 70.7	mD 50.3		mD 50.3	MD 70.7	M III 383.2	
C♭	E♭	E	F♭		F♭ 44.3	E♯ 80.6	G♭♭ 373.4	B♭♭
G♭	B♭	B	C♭		C♭ 44.9	B♯ 70.6	D♭♭ 393.7	F♭
D♭	F	F♯	G♭		G♭ 28.2	F♯♯ 73.6	A♭♭ 377.6	C♭
A♭	C	C♯	D♭		D♭ 28.8	C♯♯ 73.7	E♭♭ 396	G♭
E♭	G	G♯	A♭		A♭ 23.6	G♯♯ 73.8	B♭♭ 398.8	D♭
B♭	D	D♯	E♭		E♭ 24.8	D♯♯ 69.2	F♭ 400.9	A♭
F	A	A♯	B♭		B♭ 29.3	A♯♯ 73.4	C♭ 397.3	E♭
C	E	E♯	F		F 28.1	G♭♭ 86.5	G♭ 376.7	B♭
G	B	B♯	C		C 19	D♭♭ 106.6	D♭ 383.9	F
D	F♯	F♯♯	G		G 26.5	A♭♭ 87.7	A♭ 383.5	C
A	C♯	C♯♯	D		D 25.5	E♭♭ 90.6	E♭ 380.7	G
E	G♯	G♯♯	A		A 26.7	B♭♭ 89.8	B♭ 394.9	D
B	D♯	D♯♯	E		E 27	F♭ 96.8	F 374.8	A
F♯	A♯	A♯♯	B		B 23.2	C♭ 93.6	C 385.7	E
C♯	377.4 E♯	80.6 G♭♭	43.6 F♯		F♯	G♭	G	B
G♯	383.5 B♯	70.6 D♭♭	60.6 C♯		C♯	D♭	D	F♯
D♯	387.1 F♯♯	73.6 A♭♭	39 G♯		G♯	A♭	A	C♯
A♯	393.6 C♯♯	73.7 E♭♭	37.1 D♯		D♯	E♭	E	G♯
E♯	380.2 G♯♯	73.8 B♭♭	52.5 A♯		A♯	B♭	B	D♯
B♯	381.3 D♯♯	69.2 F♭	44.3 E♯		E♯	F	F♯	A♯
F♯♯	377.8 A♯♯	73.4 C♭	44.9 B♯		B♯	C	C♯	E♯

Table 7/3. Ptolemy-Vicentino: enharmonic tetrachords, presentation by Ordine

1548 Zarlino harpsichord range G♭ - A♯♯ with 2/7 comma tuning

19 tetrachords are correct, 5 are mistuned, 18 are impossible
(5 fewer than with Zarlino's enharmonic: see Table 6/1)
(impossible or mistuned tetrachords are greyed out)

KEY
oversize intervals (cents)
undersize intervals (cents)
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		MD 70.7	mD 50.3		mD 50.3	MD 70.7	M III 383.2	
C♭	E♭	E	F♭		F♭ -30	E♯ +30	G♭♭	B♭♭
G♭	B♭	B	C♭		C♭ -30	B♯ +30	D♭♭	F♭
D♭	F	F♯	G♭		G♭ -30	F♯♯ +30	A♭♭	C♭
A♭	C	C♯	D♭		D♭ -30	C♯♯ +30	E♭♭	G♭
E♭	G	G♯	A♭		A♭ -30	G♯♯ +30	B♭♭	D♭
B♭	D	D♯	E♭		E♭ -30	D♯♯ +30	F♭	A♭
F	A	A♯	B♭		B♭ -30	A♯♯ +30	C♭	E♭
C	E	E♯	F		F	G♭♭	G♭	B♭
G	B	B♯	C		C	D♭♭	D♭	F
D	F♯	F♯♯	G		G	A♭♭	A♭	C
A	C♯	C♯♯	D		D	E♭♭	E♭	G
E	G♯	G♯♯	A		A	B♭♭	B♭	D
B	D♯	D♯♯	E		E	F♭	F	A
F♯	A♯	A♯♯	B		B	C♭	C	E
C♯	E♯	+30 G♭♭	-30 F♯		F♯	G♭	G	B
G♯	B♯	+30 D♭♭	-30 C♯		C♯	D♭	D	F♯
D♯	F♯♯	+30 A♭♭	-30 G♯		G♯	A♭	A	C♯
A♯	C♯♯	+30 E♭♭	-30 D♯		D♯	E♭	E	G♯
E♯	G♯♯	+30 B♭♭	-30 A♯		A♯	B♭	B	D♯
B♯	D♯♯	+30 F♭	-30 E♯		E♯	F	F♯	A♯
F♯♯	A♯♯	+30 C♭	-30 B♯		B♯	C	C♯	E♯

Table 7/4. Ptolemy-Vicentino: enharmonic tetrachords, presentation by Ordine

1601 Trasuntino range F♭ - B♯♯
mistuning of intervals in 2/7 comma meantone tuning
25 tetrachords are correct, 4 are mistuned, 13 are impossible
(impossible and mistuned tetrachords are greyed out)

KEY
+ oversize intervals (cents)
- undersize intervals (cents)
enharmonic notes (Zarlino's definition)

← descending					ascending →			
M III 383.2		MD 70.7	mD 50.3		mD 50.3	MD 70.7	M III 383.2	
C♭	E♭	E	F♭		F♭ -30	E♯ +30	G♭♭	B♭♭
G♭	B♭	B	C♭		C♭ -30	B♯ +30	D♭♭	F♭
D♭	F	F♯	G♭		G♭ -30	F♯♯ +30	A♭♭	C♭
A♭	C	C♯	D♭		D♭ -30	C♯♯ +30	E♭♭	G♭
E♭	G	G♯	A♭		A♭ -30	G♯♯ +30	B♭♭	D♭
B♭	D	D♯	E♭		E♭ -30	D♯♯ +30	F♭	A♭
F	A	A♯	B♭		B♭ -30	A♯♯ +30	C♭	E♭
C	E	E♯	F		F	G♭♭	G♭	B♭
G	B	B♯	C		C	D♭♭	D♭	F
D	F♯	F♯♯	G		G	A♭♭	A♭	C
A	C♯	C♯♯	D		D	E♭♭	E♭	G
E	G♯	G♯♯	A		A	B♭♭	B♭	D
B	D♯	D♯♯	E		E	F♭	F	A
F♯	A♯	A♯♯	B		B	C♭	C	E
C♯	E♯	E♯♯	F♯		F♯	G♭	G	B
G♯	B♯	B♯♯	C♯		C♯	D♭	D	F♯
D♯	F♯♯	+30 A♭♭	-30 G♯		G♯	A♭	A	C♯
A♯	C♯♯	+30 E♭♭	-30 D♯		D♯	E♭	E	G♯
E♯	G♯♯	+30 B♭♭	-30 A♯		A♯	B♭	B	D♯
B♯	D♯♯	+30 F♭	-30 E♯		E♯	F	F♯	A♯
F♯♯	A♯♯	+30 C♭	-30 B♯		B♯	C	C♯	E♯

Table 8. Just intonation and 2/7 comma meantone

interval	just intonation		2/7 comma tempering (from just intonation)	2/7 comma cents
	ratio	cents		
syntonic comma	81:80	21.51		
1/7 comma	-	-		3.07
2/7 comma	-	-		6.14
3/7 comma	-	-		9.22
4/7 comma	-	-		12.29
minor semitone	25:24	70.67	none	70.67
major semitone	16:15	111.73	+3/7 comma	120.95
minor whole tone	10:9	182.40	+3/7 comma	191.62
major whole tone	9:8	203.91	-4/7 comma	191.62
minor third	6:5	315.64	-1/7 comma	312.57
major third	5:4	386.31	-1/7 comma	383.24
fourth	4:3	498.04	+2/7 comma	504.19
fifth	3:2	701.95	-2/7 comma	695.81
minor sixth	8:5	813.69	+1/7 comma	816.75
major sixth	5:3	884.36	+1/7 comma	887.43

Table 8A. The structure of 2/7 comma meantone

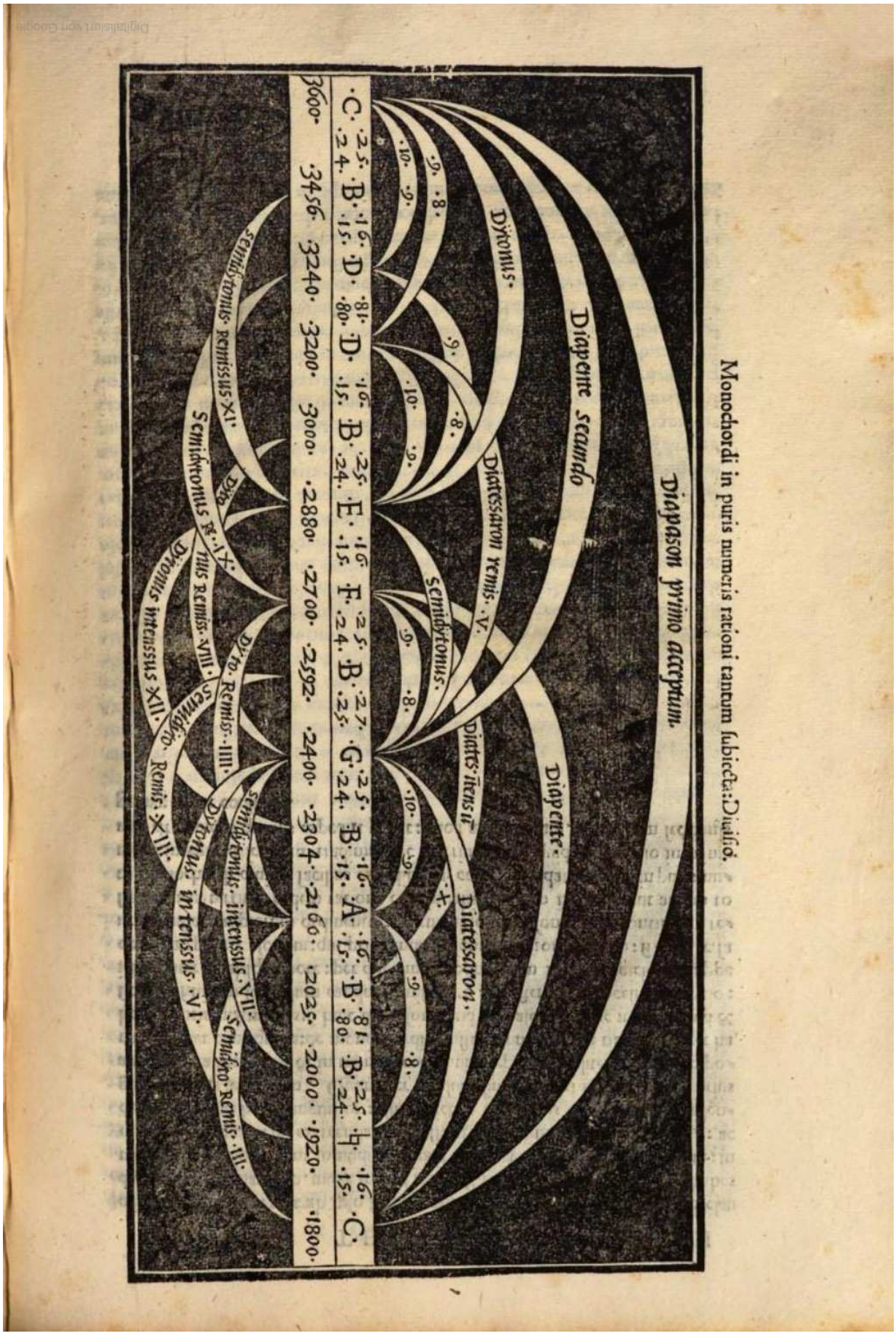
2/7 comma meantone													
	20.39		20.39			20.39		20.39			20.39		
50.28					50.28				50.28				
		50.28		50.28			50.28			50.28		50.28	
	D𝄫 50.28			C𝄪 141.34		E𝄫 241.9			D𝄪 332.96				
		C♯ 70.67	D ♭ 120.95				D♯ 262.29	E ♭ 312.57			F ♭ 433.52	E♯ 453.91	
C 0					D 191.62					E 383.24			F 504.19

Table 8B. The structure of 1/4 comma meantone

1/4 comma meantone													
	34.99		34.99			34.99		34.99			34.99		
41.06					41.06				41.06				
		41.06		41.06			41.06			41.06		41.06	
	D𝄫 41.06			C𝄪 152.10		E𝄫 234.22			D𝄪 345.26				
		C♯ 76.05	D ♭ 117.11				D♯ 269.21	E ♭ 310.26			F ♭ 427.37	E♯ 462.36	
C 0					D 193.16					E 386.31			F 503.42

This table was constructed from values to 3 decimal places, but there is one discrepancy at the second decimal place in the 41.06 values due to rounding (D𝄪 - E = 41.05).

Table 9. Fogliano's just intonation:
Musica theorica fol. XXXIIIv° (by permission of Google)



Appendix

Vito Trasuntini's letter to Camillo Gonzaga, and Vito's will

The letter written on 9th March 1607 by Vito Trasuntini to Camillo Gonzaga was found by Iain Fenlon in the Gonzaga archives, who kindly communicated this information. Elena Ghidini, the archivist in Novellara, kindly provided an image of the document from which Marco Di Pasquale made the following transcription and translation.[142]

The letter is clear evidence that Vito was still involved in affairs of the workshop even at the age of 80. As we read in the letter, he took pride in the quality of the result, although it might be reading too much into the text to infer that this implies he built all of the instrument himself.

On about the 9th October 1612 Vito contracted an illness causing "fever and catarrh".[143] His will was made on 13.10.1612, obviously in view of an impending end; Vito died the following day, aged 86.

Di Pasquale inferred that this was the only will that Vito made since there was not the conventional revoking of previous wills, such as is documented.[144] What surprised him most was that there was no mention of the workshop, tools, or instruments, from which he concluded that Vito had probably parted with these to another maker before his death.[145] By contrast Alessandro Trasuntino's will of 19.03.1542 mentioned «misier Franzesco Trasontini mio arlevo [= allievo]», which appears to describe the succession of the workshop.[146] Since the 17-year old Vito, from Monastier east of Treviso, married Laura in Venice on 13.03.1543 we can infer that he was probably already active in the Trasuntino workshop in this year.[147]

Alessando Trasuntino was credited in the Florentine archives with the manufacture of a harpsichord in 1547.[148] He died on 13.06.1552 and there is an instrument which could be attributed to Vito with the date 1552, thus it is possible that Vito was in charge of the workshop after Alessandro's death. Of the «Francesco ditto Trasontino» [...called Trasontino] we know little and cannot attribute any instruments to him.

On the basis of Di Pasquale's reasoning about Vito's will, there would have been few instruments made after 1606. In fact there is no record of any Trasuntino instrument (documentary or actual) after 1606. Additional biographical detail on Vito is included in articles by Di Pasquale.[149]

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[142] Unfortunately Marco Di Pasquale (d. 21.09 2021) did not live to see the completion of this article, although we were able to discuss some details of translation and Vito's will in November 2020. I take responsibility for the final version presented here, which is based on our discussions.
[143] Di Pasquale, 2017, p.185, note 41. «Adì 14 ottobre 1612. Misier Vido Trasuntini d'anni 86 da febre, e cattaro già cinque giorni. Medico il Vacca. Licentiato».
[144] Private communication, 9.11.2020 and Di Pasquale, 2019/2, p. 81 in the will of Antonio Ganassi.
[145] Private communication on the will, 9.11.2020.
[146] Di Pasquale, 2017, p. 184. Di Pasquale did not mention this detail in our discussions, but it is in line with his argument. Alessandro's will was published by Corsi, p. 118
[147] The wedding date is given in Di Pasquale 2019/1. Di Pasquale, 2017, p. 185 observed that Alessandro was in Treviso twice in 1543 for organ repairs, the second time in September, on which occasions he might have made Vito's acquaintance and taken him on as an apprentice. However the marriage in March 1543 in Venice suggests that the apprenticeship could already have begun early in 1543 if not 1542, an idea not discussed with Di Pasquale.
[148] See note 126.
[149] Di Pasquale, 2017 and 2019/1.

The letter to Camillo Gonzaga

Illustrissimo signor et patron colendissimo,

Già quindici giorni portai il clavicimbano al clarissimo signor Benedeto Ragazoni, ben conditionato, con sopra cassa, saratura, et tuto quello gli fa bisogno atiò lo mandase a vostra signoria illustrissima, giusto al suo ordine. Et detto signor Benedeto non [h]a voluto tenirlo in casa con dir che non vol questi intrichi. Io lo torolo a casa et son statto fin hora aspetar suo ordine né vedendo cosa alcuna ho voluto avisar vostra signoria illustrissima dil fatto et che mi comandi quello averò a fare che tanto darò esecutione ali suoi comandi. Quanto al pretio di esso, poi vostra signoria mello comise, gli scrise in [*una parola cancellata illeggibile*] schuti cento et quello piacerà a vostra signoria illustrissima che certo l'opera importa molto più come la vedrà dal efetto et bontà di esso istrumento. Et se io sapese il come mandarlo fin hora vostra signoria illustrissima l'averia auto et così starò aspetando sua comisione che di subitto lo manderò. Et per fine a vostra signoria illustrissima melle inchino et bacio le mane.

Di Venetia oggi 9 marzo 1607

Di vostra signoria illustrissima
servitor humilissimo
Vito Trasontin [*forse* Trasontini]

[*a tergo*]
Al illustrissimo signor mio signor colendissimo il signor
Camillo Gonzaga conte di
[*lacuna nella carta?*]
Racomandata a Grasin ebreo
In Mantova Nogolara

1607
[*una parola illeggibile*] Morosini Venetia 9
marzo

Most illustrious lord and most honorable patron,

Already fifteen days ago I took the harpsichord — well-furnished, with external case[150], lock, and all that is necessary — to the very distinguished Signor Benedetto Ragazzoni so that, according to your order, he sent it to your most illustrious lordship. The said Signor Benedetto did not want to keep it at home saying that he does not want inconveniences of this kind. I keep it at my house and have awaited your order so far, but not seeing anything, I want to warn your most illustrious lordship of the fact and that you command me what I have to do so that I will immediately carry out your commands. As for its price, after your lordship committed it to me, I wrote to you that it is [*a deleted word*] a hundred scudi and it will please your most illustrious lordship that certainly the work is worth much more, as you will see from the effect and quality of the said instrument. And if I knew how to send it, your most illustrious lordship would already have it, therefore I will wait for your commision and send it immediately. And to finish I bow to your most illustrious lordship and kiss your hands.

Venice, today 9 March 1607

Of your most illustrious lordship
most humble servant
Vito Trasontin [*perhaps* Trasontini][151]

[*on the back*]
To the most illustrious lord and most honorable patron,
Camillo Gonzaga count of
[*lacuna in the paper?*]
Registered to the jewish Grasin
In Mantua Nuvolara

1607
[*an illegible word*] Morosini Venice 9
March

[150] This could mean an "outer case" or a packing case". Outer cases sometimes have a lock.
[151] An ink smudge renders the last letter unclear.

Vito Trasuntini's will

Venezia, Archivio di Stato, *Notarile*, *Testamenti*, notaio Giovanni Battista Piazzola, b. 802, cedola 96.

[*conventional notarial formulas; 13 October 1612*] Considerando io Vido Trasontini del quondam Antonio li pericoli di questa fragil vitta, né cosa alcuna esser più certa della morte, né più incerta dell'hora di quella, ho voluto mentre che la raggion governa le genti disponer delle cose mie sospese. Attrovandomi sano per gratia del signor Iddio della memoria, senso et intelletto, benché del corpo infermo, giacendo in letto in casa della mia habitattion in contrà de San Paternian, ho mandato a chiamar da me Giovanni Battista Piazzuolla nodaro di Venetia, qual ho pregato che scrivi il presente mio testamento et quello dappoi la mia morte lo vogli complir et roborar giusta la forma di statuti di questa città, per el qual prima racomando l'anima mia al signor [*one word unreadable*] Iddio, alla beatissima Vergine Maria, et a tutta la corte celestial. Mi attrovo esechutor della comissaria del quondam misier Claudio mio fiol de ducati mille che mi ha lasciato per il suo testamento delli quali intendo di disponere. Lasso a mia fia natural, sia o non sia mia fia, per il suo maridar ducati seicento, lasso li restanti ducati quattrocento delli suddetti mille a Marietta che governa detta mia fiola. Voglio che detta madona Marietta sia obligata delli detti ducati 400 che li lasso de dar ducati 25 a Felicita mia masara de casa[152] senza contradition alcuna, et debba alla mia sepoltura tor il capitolo della mia contrà, et che facia lei la spesa della mia sepoltura. Lasso anco a Paula mia sorella ducati 25 delli detti ducati 400. Interrogato da me nodaro delli lochi pij, poveri, vergognosi, schiavi, Mendicanti, Pietà, Miserabelli, San Zuanepaulo, Cittelle et altri, respose non voler far altro. In nelle [*one word unreadable*] testamenti disse non voglio che alcuno sia chi si voglia possi molestar mia nuora in qual si voglia cosa della sua administratione intendendo io restar sodisfattissimo et così intendo che resti quieta et libera che alcuno la possa dimandare cosa alcuna.
Io Santo Lanza, dottor, piovano di San Paternian fui testimonio pregato, et giurato et [*one word unreadable*].
Io Is[e]po Straminati fa chas[s]e da spiegi dil quondam Beltrame fui testimoni[o] giurato et pregato.

[152] A note by Marco Di Pasquale, (private communication, 9.11.2020): Masara de casa (modern Italian, though with wider meaning: massaia) = housekeeper. The most reliable Veneziano-Italiano dictionary is that by Giuseppe Boerio.

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https://books.google.de/books/about/De_tutte_l_opere_del_r_m_Gioseffo_Zarlin.html?id=uFuYojvNd1gC&redir_esc=y

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https://books.google.de/books/about/Sopplimenti_musicali_etc.html?id=VH4Iq2Vhx5wC&redir_esc=y
[N.B. The varied spelling of the Zarlino titles is correctly given for the different editions]